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Designs of Wideband 3dB Branch-line Couplers for ALMA Bands 3 to 10

Hiroya Andoh, Tetsuhiro Minamidani*, Shin'ichiro Asayama†,
Yoshinori Yonekura†, Hideo Ogawa†,
Norikazu Mizuno*, Akira Mizuno*and Yasuo Fukui*

Toyota National College of Technology
2-1 Eisei-cho, Toyota, Aichi 471-8525, Japan
E-mail: andoh@toyota-ct.ac.jp

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Abstract

A guiding principle of design of wideband waveguide 3-dB 5-branch-line couplers for ALMA bands 3 to 10 in suitable for use in SSB heterodyne receivers, their designs and their performances are reported. The couplers have good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89-91 degrees over the frequency range of each ALMA band. The couplers for the ALMA band 3, 4, 5, 6, 7, 8, 9 and 10 include WR-10, WR-6, WR-5, WR-3, WR-2.8, WR-2.2, WR-1.5 and WR-1.2 waveguides, respectively.

The couplers are designed by a numerical analytical method by using matrices based on the circuit theory and they are optimized with 3D EM simulator (HFSS). The numerical analytical method for seeking S-matrices mentioned in this memo enables quick calculations for performances of 3-dB branch-line couplers in the case of optimization and possesses advantages over electromagnetic simulation software in common use.

Keywords

ALMA bands 3 to 10, wideband waveguide quadrature hybrid, wideband waveguide 3dB 5-branch-line coupler, numerical matrix analysis, S-matrix

1 Introduction

Split-block branch-line couplers with strong coupling (e.g.,3 dB) are easily made and has already reported [1], however, no clear guiding principles to design wideband split-block branch-line couplers have been presented. Authors have already reported a wideband waveguide 3-dB 5-branch-line coupler suitable for use in SSB heterodyne receivers in ALMA Band 3 (84-116 GHz) [2][3]. The coupler is designed by a numerical analytical method by using matrices based on the circuit theory and has good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89.5-90.7 degrees over the frequency range of 84GHz-116GHz.

In this manuscript, a guiding principle to design wideband waveguide 5-branch-line couplers for ALMA bands 3 to 10, their designs and their performances are reported. Initially, the couplers are designed by a numerical analytical method by using matrices based on the circuit theory and they are optimized with

*Nagoya University, Furo-cho, Chigusa-ku, Nagoya, Aichi 464-8601, Japan
†Osaka Prefecture University, 1-1 Gakuen-cho, Sakai, Osaka 599-8531, Japan

Table 1: Waveguides for ALMA bands

Band	from - to (GHz)	f_0 (GHz)	Δf (GHz)	$\frac{\Delta f}{f_0}$	EIA (VDI)	a (mm)	b (mm)	f_c (GHz)
3	84 - 116	100	32	32.0%	WR-10	2.54	1.27	59.1
4	125 - 163	144	38	26.4%	WR-6	1.651	8.255	90.9
5	163 - 211	187	48	25.7%	WR-5	1.295	0.6477	115.8
6	211 - 275	243	64	26.3%	WR-3	0.8636	0.4318	173.7
7	275 - 370	322.5	95	29.5%	WR-2.8	0.7112	0.3556	210.9
8	385 - 500	442.5	115	26.0%	WR-2.2	0.5588	0.2794	268.4
9	602 - 720	661	118	17.9%	WR-1.5	0.3810	0.1905	393.7
10	787 - 950	868.5	163	18.8%	WR-1.2	0.3048	0.1524	492.1

3D EM simulator (HFSS). Being worked toward practical use, the designs for the ALMA bands 3, 4, 5, 6, 7, 8, 9 and 10 include WR-10, WR-6, WR-5, WR-3, WR-2.8, WR-2.2, WR-1.5 and WR-1.2 waveguides as shown in Table 1, respectively.

2 A guiding principle to design wideband waveguide 3-dB couplers

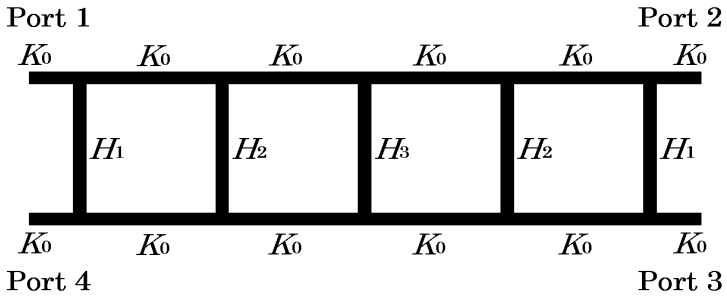

Figure 1: A schematic of 5-branch-line coupler with 10 E-plane T-junctions.

A schematic of 5-branch-line coupler with 10 E-plane T-junctions is shown in Fig.1, including two waveguides with the characteristic impedance K_0. The characteristic impedances of slits between the two waveguides are H_1, H_2, H_3, H_2 and H_1, respectively.

The S-matrix of waveguide 3-dB 5-branch-line coupler with E-plane junctions as shown in Fig.1 is as follows[2].

$$\left(\begin{array}{ccccc|ccccc} S_{11} & \star & \star & \star & S_{12} & S_{13} & \star & \star & \star & S_{14} \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star \\ S_{21} & \star & \star & \star & S_{22} & S_{23} & \star & \star & \star & S_{24} \\ \hline S_{31} & \star & \star & \star & S_{32} & S_{33} & \star & \star & \star & S_{34} \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star \\ S_{41} & \star & \star & \star & S_{42} & S_{43} & \star & \star & \star & S_{44} \end{array}\right) = 2 \left(\begin{array}{ccccc|ccccc} \frac{1+H_1}{j\tan\theta}+1 & \frac{1}{j\sin\theta} & 0 & 0 & 0 \\ \frac{1}{j\sin\theta} & \frac{2+H_2}{j\tan\theta} & \frac{1}{j\sin\theta} & 0 & 0 \\ 0 & \frac{1}{j\sin\theta} & \frac{2+H_3}{j\tan\theta} & \frac{1}{j\sin\theta} & 0 \\ 0 & 0 & \frac{1}{j\sin\theta} & \frac{2+H_2}{j\tan\theta} & \frac{1}{j\sin\theta} \\ 0 & 0 & 0 & \frac{1}{j\sin\theta} & \frac{1+H_1}{j\tan\theta}+1 \\ \hline 0 & 0 & 0 & 0 & \frac{H_1}{j\sin\theta} \\ 0 & 0 & 0 & \frac{H_2}{j\sin\theta} & 0 \\ 0 & 0 & \frac{H_3}{j\sin\theta} & 0 & 0 \\ 0 & \frac{H_2}{j\sin\theta} & 0 & 0 & 0 \\ \frac{H_1}{j\sin\theta} & 0 & 0 & 0 & 0 \end{array}\right)$$

$$
\left(\begin{array}{ccccc|ccccc}
0 & 0 & 0 & 0 & \frac{H_1}{j\sin\theta} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{H_2}{j\sin\theta} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{H_3}{j\sin\theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{H_2}{j\sin\theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{H_1}{j\sin\theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
\frac{1+H_1}{j\tan\theta}+1 & \frac{1}{j\sin\theta} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\frac{1}{j\sin\theta} & \frac{2+H_2}{j\tan\theta} & \frac{1}{j\sin\theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{j\sin\theta} & \frac{2+H_3}{j\tan\theta} & \frac{1}{j\sin\theta} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{j\sin\theta} & \frac{2+H_2}{j\tan\theta} & \frac{1}{j\sin\theta} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{j\sin\theta} & \frac{1+H_1}{j\tan\theta}+1 & 0 & 0 & 0 & 0 & 1
\end{array}\right)^{-1}
\cdot
\left(\begin{array}{ccccc|ccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right)
$$

$$
-
\left(\begin{array}{ccccc|ccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right) \quad (1)
$$

In the above equation, the value of H_1, H_2 and H_3 are normalized by K_0 ($H_1 \leftarrow \frac{H_1}{K_0}$, $H_2 \leftarrow \frac{H_2}{K_0}$, $H_3 \leftarrow \frac{H_3}{K_0}$), and the notes of "$\star$" in the matrix do not have to be taken care of. The quantity θ represents the electrical "length" of each transmission line. Here, the following conditions have to be satisfied for 3-dB coupling at center frequency. (Similar formulas for branch-line couplers up to six branches are shown in [4].)

$$H_2 = \frac{1}{H_1 + \sqrt{2} + 1} \quad (2)$$

$$H_3 = \frac{1 - 2H_1 - {H_1}^2}{\sqrt{2}} - \frac{(|S_{21}| - |S_{31}|)_{@f=f_0}}{18.6} \quad (3)$$

Especially, in the case of TE_{10} waveguide branch-line couplers, the frequency dependence of θ is;

$$\theta(x) = \frac{\pi}{2}\sqrt{\frac{x^2 - \alpha^2}{1 - \alpha^2}}, \quad (4)$$

where $x = f/f_0$ and $\alpha = f_c/f_0$. The value of f, f_0 and f_c are the frequency of the applied signal, the center frequency and the cut-off frequency of the waveguide, respectively.

Figure 2 and 3 show the performances of a wideband 3dB 5-branch-line coupler. In this case, the values of H_1, H_2 and H_3 are 0.222, 0.326($= \frac{1}{H_1+\sqrt{2}+1} - \frac{1}{18.6}$) and 0.358($= \frac{1-2H_1-{H_1}^2}{\sqrt{2}}$), respectively. The value of α is 0.59. This is the value of cut-off frequency f_c for WR-10 waveguide (2.54mm×1.27mm) to the center frequency of f_0=100GHz. As you can see in this figure, this coupler have good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89-91 degrees over the frequency range of $0.8f_0 < f < 1.2f_0$. Thus, we decided the guiding principle of design of wideband waveguide 3-dB couplers for all ALMA bands as stated above.

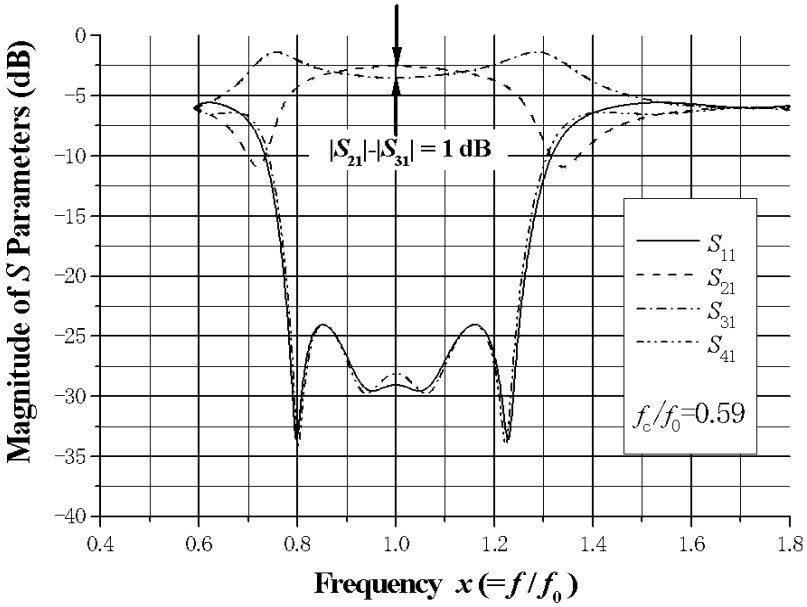

Figure 2: Numerical results for the frequency dependence of S parameters of 3dB 5-branch-line coupler in the case of $H_1 = 0.222$ and $|S_{21}| - |S_{31}|$=1dB at the center frequency ($H_2 = 0.326$ and $H_3 = 0.358$). The value of $\alpha = f_c/f_o$ is 0.59.

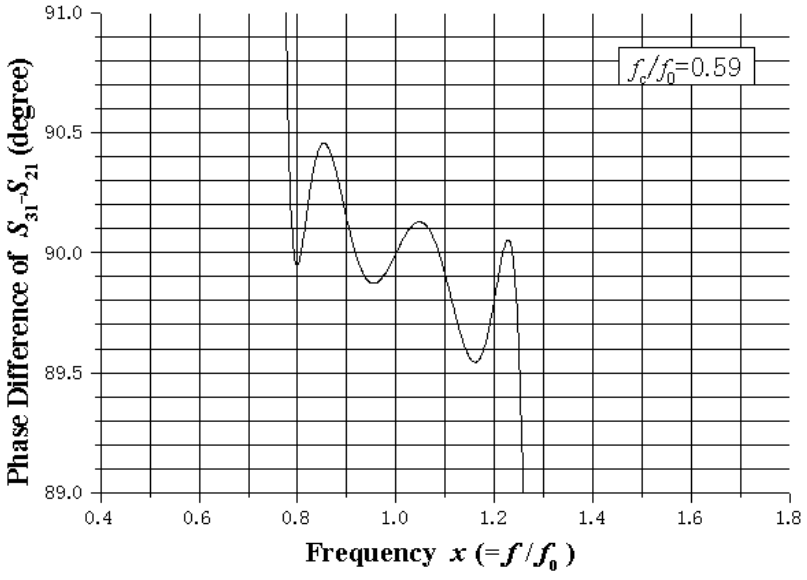

Figure 3: Numerical results for the frequency dependence of a phase difference of 3dB 5-branch-line coupler in the case of $H_1 = 0.222$ and $|S_{21}| - |S_{31}|$=1dB at the center frequency ($H_2 = 0.326$ and $H_3 = 0.358$). The value of $\alpha = f_c/f_o$ is 0.59.

3 Practical designs of 3-dB couplers for ALMA bands 3 to 10

Figures 4, 5, 6, 7, 8, 9, 10 and 11 show the designs of the 3-dB couplers for the ALMA band 3, 4, 5, 6, 7, 8, 9 and 10, respectively. They include WR-10, WR-6, WR-5, WR-3, WR-2.8, WR-2.2, WR-1.5 and WR-1.2 waveguides, respectively. The couplers in the figures have good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89-91 degrees over the frequency range of each ALMA band.

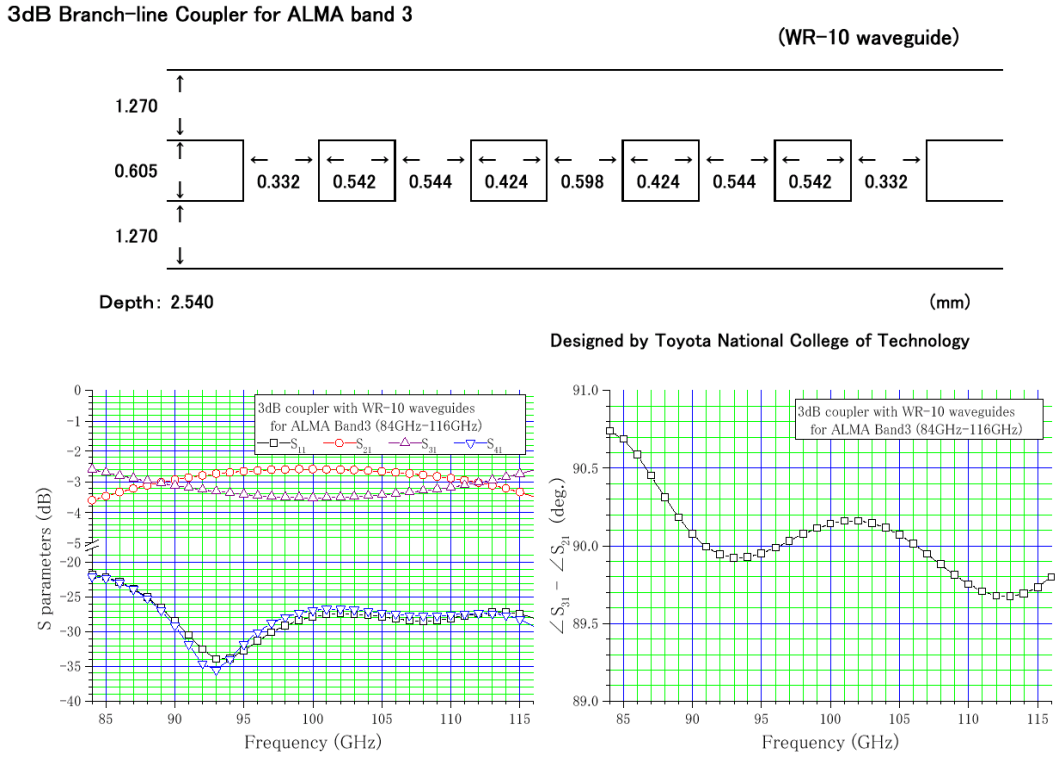

Figure 4: The design and the performances of a waveguide 3-dB coupler for ALMA band 3

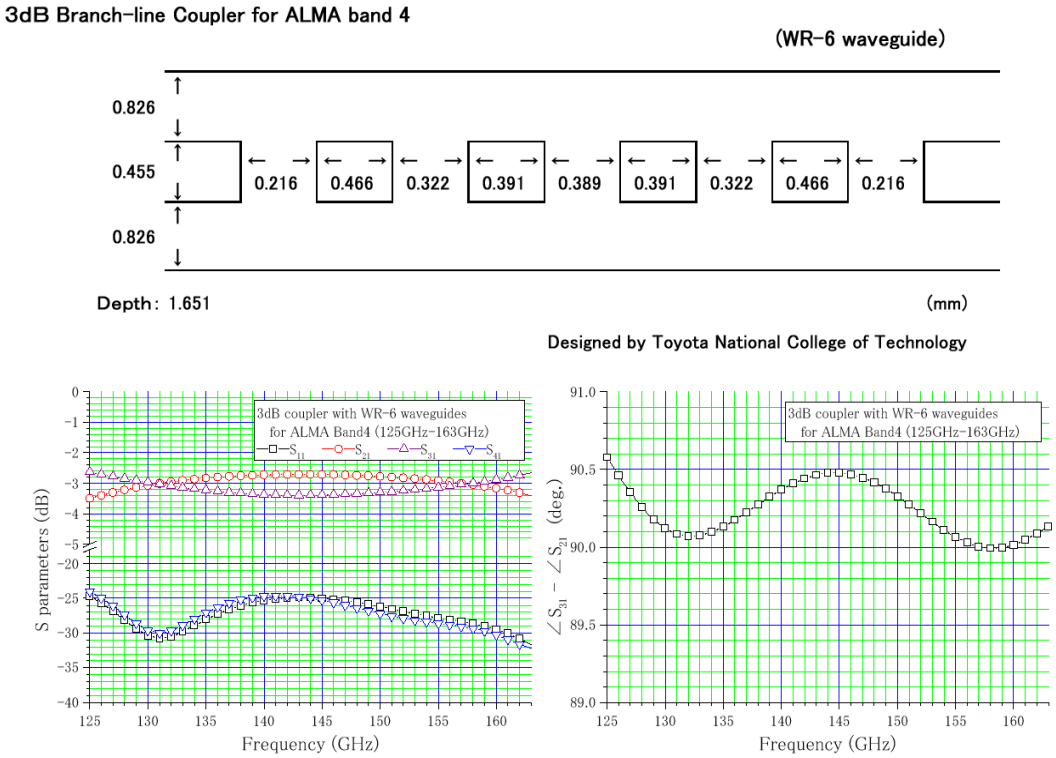

Figure 5: The design and the performances of a waveguide 3-dB coupler for ALMA band 4

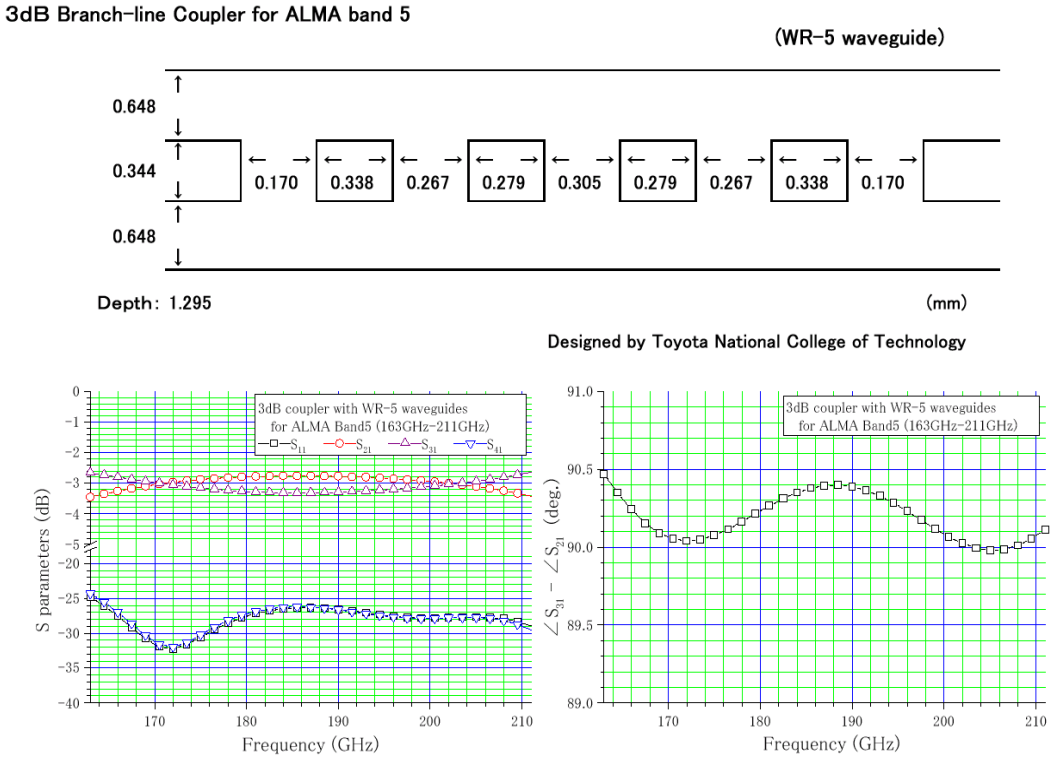

Figure 6: The design and the performances of a waveguide 3-dB coupler for ALMA band 5

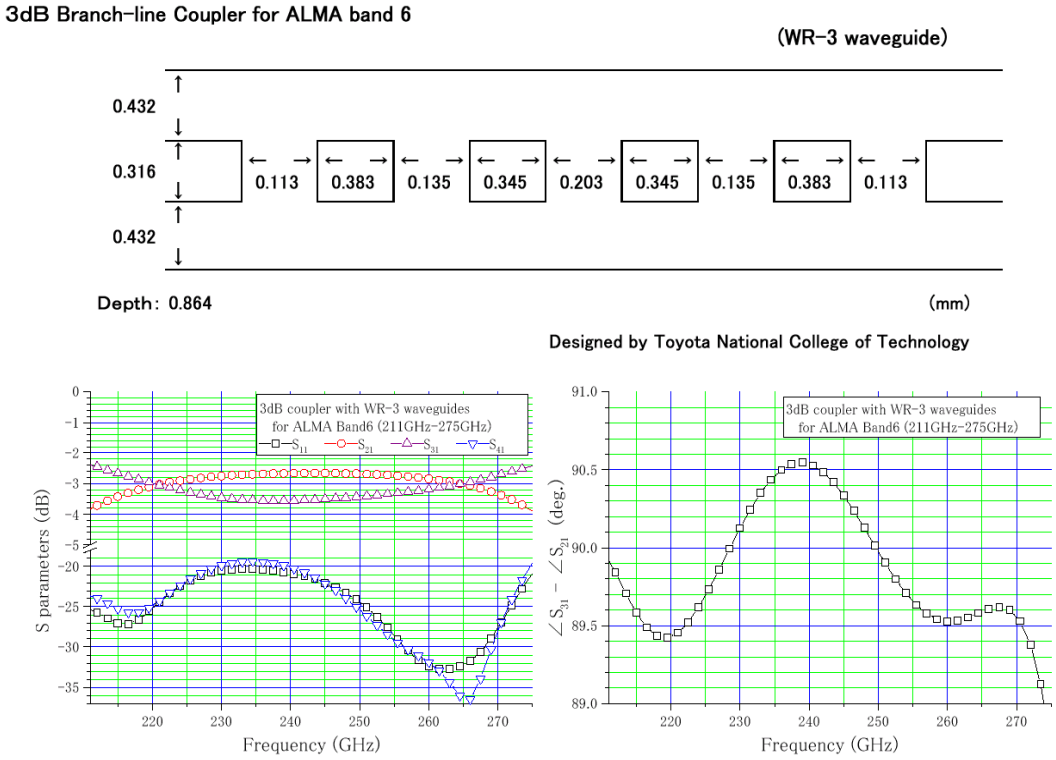

Figure 7: The design and the performances of a waveguide 3-dB coupler for ALMA band 6

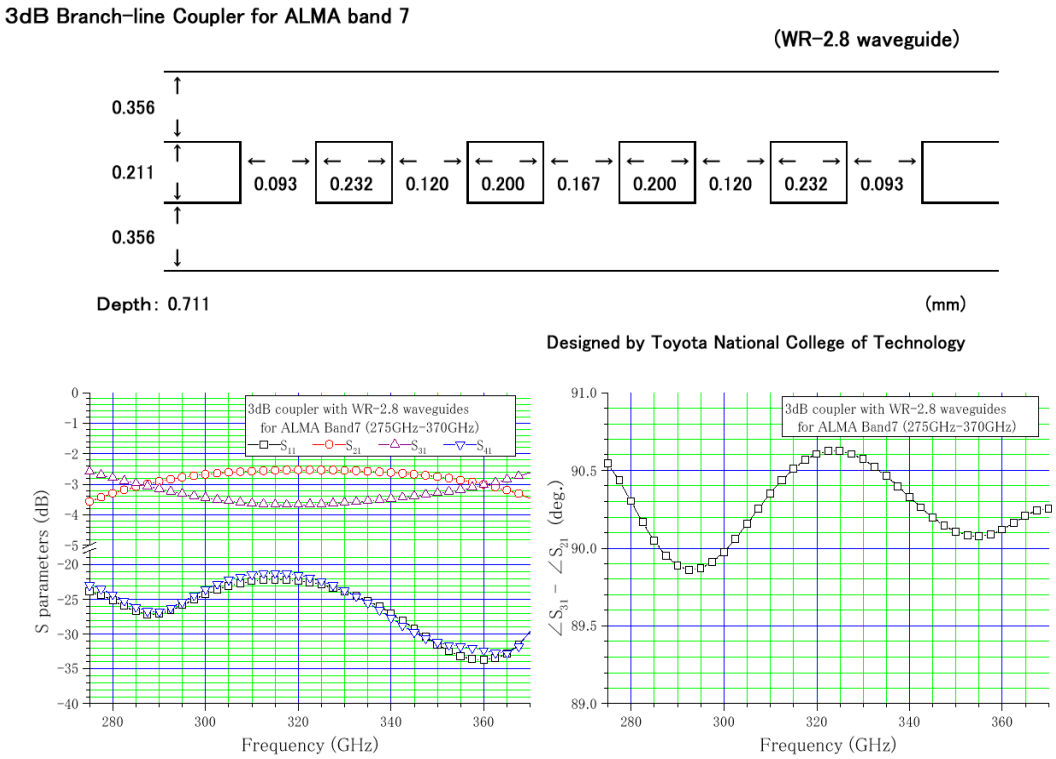

Figure 8: The design and the performances of a waveguide 3-dB coupler for ALMA band 7

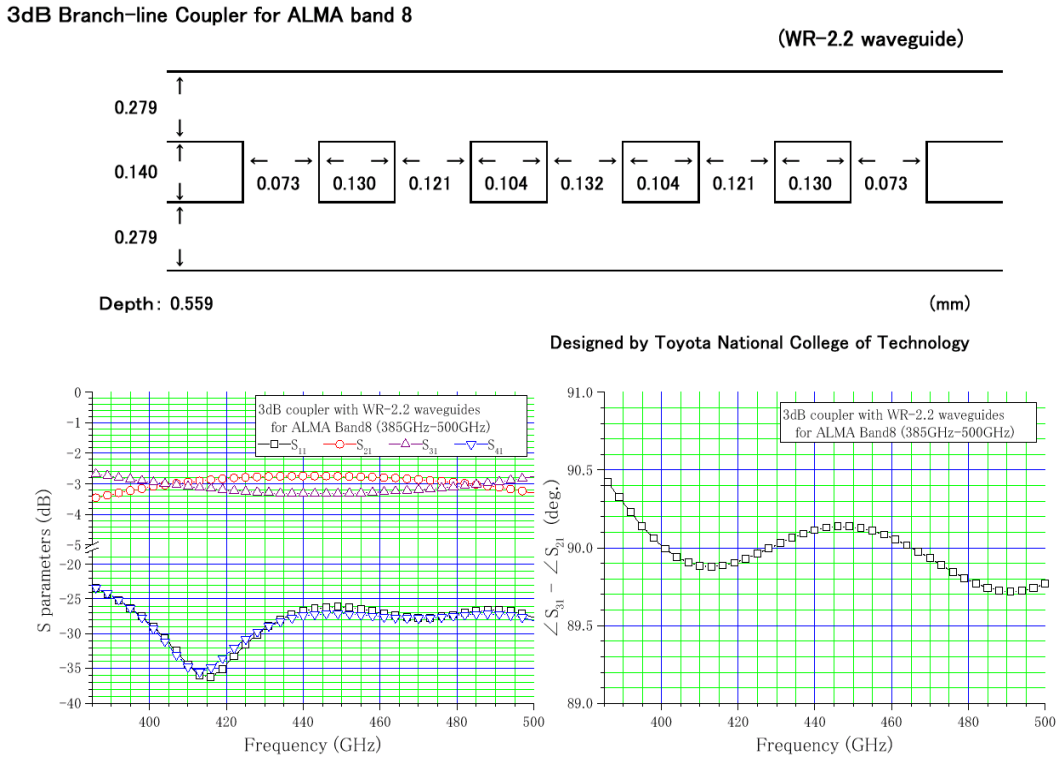

Figure 9: The design and the performances of a waveguide 3-dB coupler for ALMA band 8

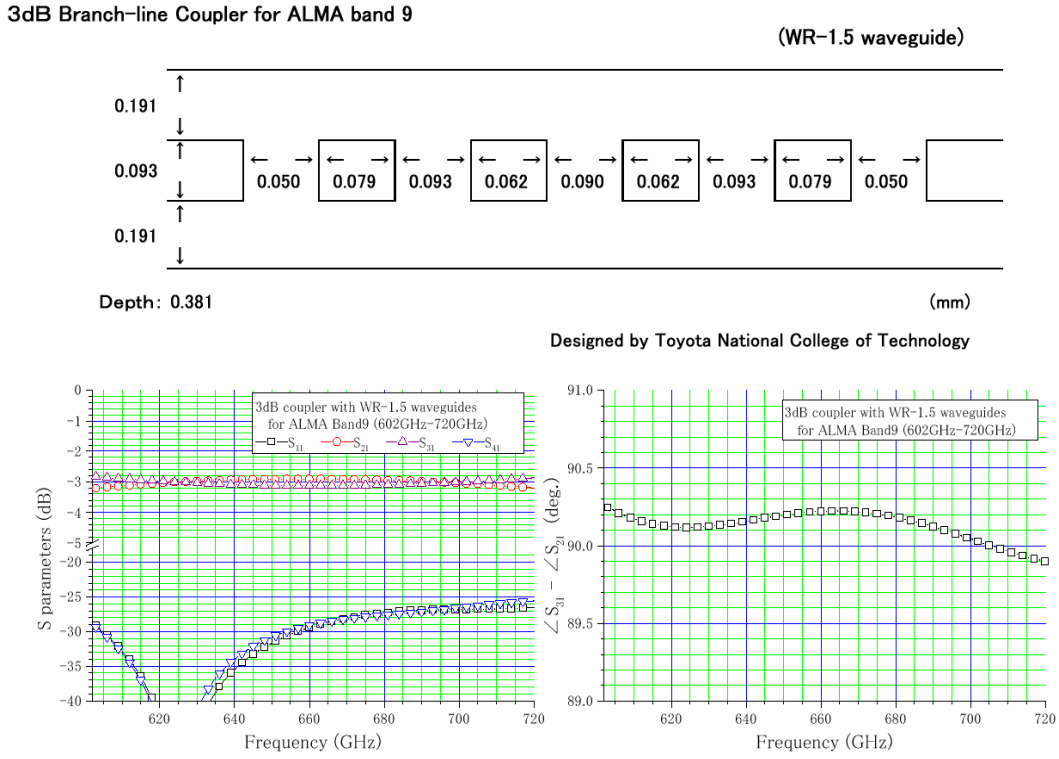

Figure 10: The design and the performances of a waveguide 3-dB coupler for ALMA band 9

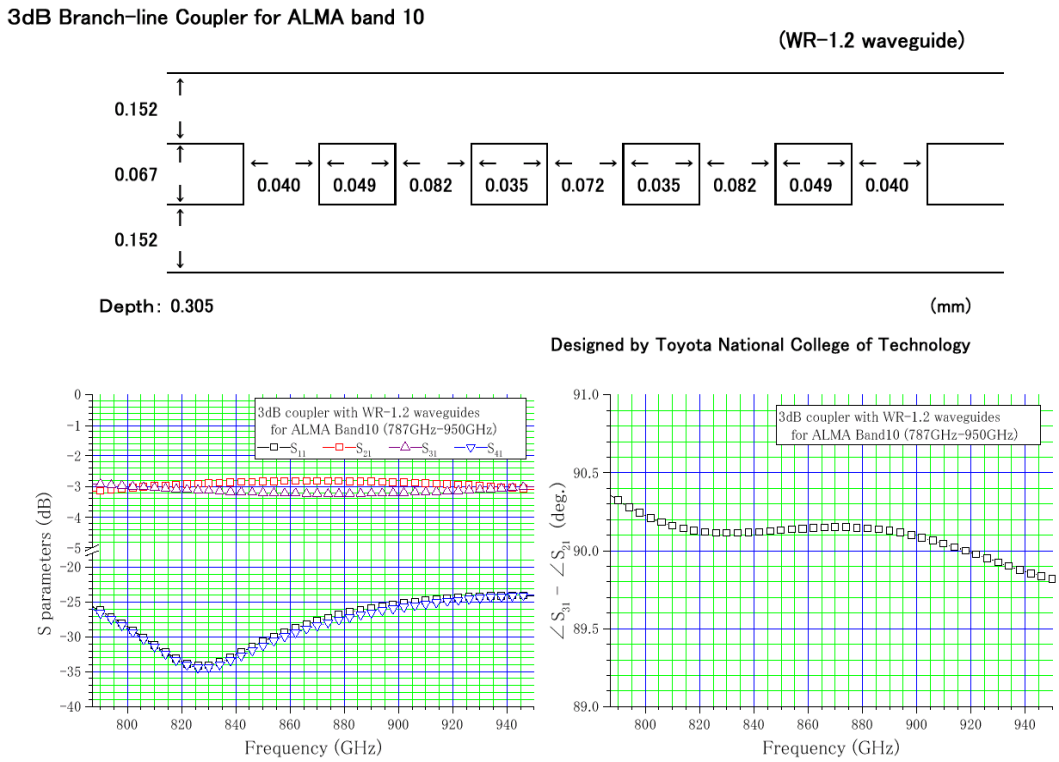

Figure 11: The design and the performances of a waveguide 3-dB coupler for ALMA band 10

4 Summary

We presented a guiding principle of design of wideband waveguide 3-dB 5-branch-line couplers for ALMA bands 3 to 10 in suitable for use in SSB heterodyne receivers and also showed the designs and the performances. The couplers have good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89-91 degrees over the frequency range of each ALMA band.

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