The intensity calibration for HIFI

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Abstract

Existing schemes for the calibration of radio-astronomical observations are insufficient for the application in forthcoming space missions with heterodyne systems like HIFI. Many of the approximations used are not valid for systems with large IF frequencies and they are too rough for the desired calibration accuracy of a few percent. Moreover, they do not exploit the full capabilities of instruments with two thermal load for the bandpass calibration. We propose a new calibration scheme for the planning and reduction of HIFI observations, taking advantage of the lack of an atmosphere and correcting for the effects of standing waves in the combined observation of lines and continuum with HIFI.

The new calibration scheme uses a separate OFF measurement to determine the properties of standing waves between the subreflector and the receiver. Two different effects are corrected. Standing wave ripples are removed from the continuum level providing the spectral baseline of the observations and the modulation of the line strengths by standing waves is corrected in the absolute line calibration. However, at high frequencies the standing wave correction imposes severe constraints on the integration time for the OFF measurement. A reasonable use of the standing-wave calibration is thus only possible if the standing wave pattern is stable over long time scales.

We have also provided an accurate estimate of the error budget of the calibration allowing to put clear limits on the accuracy of the different instrumental parameters required to achieve a given intensity calibration accuracy.

Contents

1 Introduction

The standard schemes for the calibration of spectroscopic measurements in radio-astronomy as summarised e.g. by Kutner & Ulich (1981), Downes (1988), Hiyama (1998), or Mangum (2002) are not the most appropriate concept for the intensity calibration of space mission data as expected from HIFI. They were mainly set up to deal with the instability of the atmosphere, implicitly assume a low intermediate frequency, and do not contain any terms for the description of standing waves in the optical path. Thus a new calibration scheme is necessary for HIFI. Instead of incorporating the usually used quantities into a new, more complex calibration scheme we develop a new scheme from the first principles which is optimised to describe the conditions for HIFI without sticking to the previously used nomenclature.

2 System response theory

2.1 The system response in an astronomical observation

The new scheme has to take into account that in the double-sideband design for HIFI it is not guaranteed that any quantity is the same in both sidebands. Thus the split into both sideband contributions should be done already on the level of the bandpass. In this section a linear system response is assumed so that the different contributions can be added up. Modifications due to nonlinearity are discussed in Sect. 5.

For the observation of an astronomical source the count rate at the backends c is determined by:

$$\begin{aligned} c \;=\; & \gamma_{\rm ssb}\left\{\eta_{\rm l,ssb}\left[\eta_{\rm sf,ssb}J_{\rm S,ssb}+(1-\eta_{\rm sf,ssb})J_{\rm R,ssb}\right]+(1-\eta_{\rm l,ssb})J_{\rm T,ssb}\right\} \\ & +\gamma_{\rm isb}\left\{\eta_{\rm l,isb}\left[\eta_{\rm sf,isb}J_{\rm S,isb}+(1-\eta_{\rm sf,ssb})J_{\rm R,isb}\right]+(1-\eta_{\rm l,isb})J_{\rm T,isb}\right\} \\ & +\gamma_{\rm rec}J_{\rm rec}+z \end{aligned} \tag{1}$$

Eq. (1) holds independently for each backend channel. Consequently, all calibration factors may as well depend on the considered channel. Hence, a complete characterisation of the instrument would need $(3\times 2+3)\times n$ quantities when n backend channels are considered.

The quantities $\gamma_{\rm ssb}$ and $\gamma_{\rm isb}$ are the bandpasses in the signal and the image sideband providing the translation factor from radiation intensities to spectrometer counts. In this notation they include all contributions from the input conversion factor g as used by Kutner & Ulich (1981) and from the gain ratios $G_{\rm ssb}$ and $G_{\rm isb}$ as used by Ossenkopf (2002). In the following, all quantities are allowed to differ between the signal sideband (ssb) and the image sideband (isb).

The coupling of the different radiation fields to the total field in the beam are described by efficiencies η. We treat them pairwise where one η gives to coupling to a particular input contribution and $(1-\eta)$ is the remaining coupling to all other contributions. The quantity $\eta_{\rm l}$ is the effective forward efficiency, i.e. the part of the total beam that actually reaches the sky in the considered sideband. It contains the main beam, error beam contributions, scattering, and spill-over that terminate on the sky. Consequently $(1-\eta_{\rm l})$ is the part of the beam that terminates somewhere on the warm telescope structure. This includes the ohmic losses at the telescope surface which are visible as warm radiation. Contributions from the cold structure in the focal plane unit are treated as part of the receiver temperature as discussed below.

$\eta_{\rm sf}$ is the source efficiency with respect to the forward beam, i.e. the contribution of the sky beam that is filled by source emission. For a known source structure it can be computed by

$$\eta_{\rm sf} = \eta_{\rm sf}(\Omega) = \iint_{\rm sky} B_{\rm n}(\Psi) P_{\rm n}(\Psi - \Omega) d\Psi \Big/ \iint_{\rm sky} P_{\rm n}(\Omega) d\Omega \tag{2}$$

where, $P_{\rm n}(\Omega)$ is the antenna power pattern, and $B_{\rm n}(\Omega)$ is the brightness distribution of the astronomical source on the sky normalised to a maximum value of 1. The quantity $\eta_{\rm sf}$ corresponds to $\eta_{\rm c}\eta_{\rm fss}$ in the notation of Kutner & Ulich (1981). For a Gaussian source brightness distribution and a perfect Gaussian beam it is given by $\eta_{\rm sf} = \theta^2_{\rm source}/(\theta^2_{\rm source} + \theta^2_{\rm beam})$. The determination of $\eta_{\rm sf}$ is discussed in a separate paper (Kramer in prep.).

The quantities $J_{\rm S}$, $J_{\rm R}$, and $J_{\rm T}$ denote the radiation temperatures from the source, the blank sky, and the sum of the telescope contributions within the beam. In case of black body contributions with a single temperature the radiation fields can be computed using the Planck radiation formula:

$$J_{\rm B} = B_\nu(T) = \frac{2h\nu^3}{c^2} \left[\exp\left(\frac{h\nu}{kT}\right) - 1 \right]^{-1} \tag{3}$$

It is essential to take the full frequency dependence into account here because coherent double-sideband receivers are able to distinguish between the energies of photons coming from the different sidebands. Instead of using an intensity scale, the radiation field can be expressed in terms of a radiation temperature by the translation into an equivalent Rayleigh-Jeans temperature. If we perform this transformation for the LO frequency

$$J = \frac{c^2}{2k\nu^2_{\rm LO}} J_{\rm B} \tag{4}$$

we obtain a measure for the radiation field temperature, which has the correct dependence of the photon energy on the IF frequency but uses the familiar temperature scale to characterise the absolute value of the radiation field. In the following we will thus use the so defined radiation field J when providing numbers for the different contributions. In general, however, the radiation fields will be a superposition of contributions from several temperatures. In contrast to ground-based observations where the telescope structure is in thermal equilibrium with the ambient temperature we cannot even expect an accurate knowledge of $J_{\rm T}$ for HIFI, so that the absolute value of $J_{\rm T}$ introduces a considerable uncertainty. As discussed by Ossenkopf (2002) the contribution of the radiation field from the blank sky can be neglected in most cases, i.e. $J_{\rm R} = 0$.

Eq. (3) shows that even a single temperature radiator results in different radiation levels in both sidebands of the receiver. Although the intermediate frequency of HIFI is so large that the sideband imbalance of any radiation has to be taken into account, it is possible to use a linear description for the frequency dependence of the radiation field from the continuum sources. A linear expansion of the radiation field around the LO frequency shows an accuracy better than 10^{-3} within the whole IF range. We can write

$$J_{\nu_{\rm USB}} - J_{\nu_{\rm LO}} = J_{\nu_{\rm LO}} - J_{\nu_{\rm LSB}} = b\nu_{\rm IF} \times J_{\nu_{\rm LO}} \tag{5}$$

In the Rayleigh limit the factor $b = 2/\nu_{\rm LO}$ is independent of the actual temperature, so that changes in the temperature are not reflected in the frequency dependence of the radiation. An exact derivation taking the curvature of the Planck curve into account results in $b_{\rm h} \approx 2.02/\nu_{\rm LO} - 0.00028/{\rm GHz}$ at the 100 K hot load, $b_{\rm T} \approx 2.03/\nu_{\rm LO} - 0.00036/{\rm GHz}$ for a telescope temperature of 80 K, and $b_{\rm c} \approx 2.50/\nu_{\rm LO} - 0.00029/{\rm GHz}$ at the 15 K cold load.

The radiation field $J_{\rm rec}$ characterises the receiver temperature. It also includes contributions from surfaces at the temperature of the focal plane unit (FPU) in the optical path. This approach is justified when the contributions do not differ between the source and the reference or load measurements and the FPU temperature remains stable. Otherwise the corresponding radiation field should be added in Eq. (1) as an additive term for both sidebands. The receiver temperature $J_{\rm rec}$ cannot be split into contributions from both sidebands. In all measurements only the combined quantity $\gamma_{\rm rec} J_{\rm rec}$ is obtained. A priory there is no simple relation between $\gamma_{\rm rec}$ and the bandpasses for the sky and telescope contributions $\gamma_{\rm ssb}$ and $\gamma_{\rm isb}$. But as $\gamma_{\rm rec}$ cannot be determined independently of $J_{\rm rec}$ we are free to chose any physically reasonable definition for $\gamma_{\rm rec}$, e.g. $\gamma_{\rm rec} = \gamma_{\rm ssb} + \gamma_{\rm isb}$, thus only redefining the receiver temperature $J_{\rm rec}$. The quantity z represents the zero counts of the backend, which can be easily measured when terminating the backend input.

Eq. (1) holds independently for each backend channel. Consequently, all calibration factors may as well depend on the considered channel. Hence, a complete characterisation of the instrument would need $(3 \times 2 + 3) \times n$ quantities when n backend channels are considered.

2.2 The sky reference

Most observations will be calibrated using the observation of a point free of emission as a reference. If the optical path is the same for the source observation and the reference observation we can expect the same system response in both measurements. Then the count rate at the blank sky position is given by:

$$\begin{aligned} c_{\rm OFF} \;=\; & \gamma_{\rm ssb}\left\{\eta_{\rm l,ssb} J_{\rm R,ssb} + (1-\eta_{\rm l,ssb}) J_{\rm T,ssb}\right\} \\ & +\gamma_{\rm isb}\left\{\eta_{\rm l,isb} J_{\rm R,isb} + (1-\eta_{\rm l,isb}) J_{\rm T,isb}\right\} \\ & +\gamma_{\rm rec} J_{\rm rec} + z \end{aligned} \tag{6}$$

As a slew to a reference position is relatively slow, a focal plane chopper allows to quickly point towards a nearby reference position by slightly changing the optical path. However, this has the side effect that the bandpass and the coupling efficiencies may deviate between the two positions. For a chopped observation, Eq. (6) has thus to be rewritten by changing all coefficients to corresponding quantities for the second chop position, i.e. $\gamma_{\rm ssb} \to \gamma^{\rm R}_{\rm ssb}$, $\eta_{\rm l,ssb} \to \eta^{\rm R}_{\rm l,ssb}$ and so on.

2.3 Load measurements

A basic calibration of the instrument is performed by a pair of measurements on two well defined thermal sources in the focal plane unit (FPU). This calibration by two thermal loads is typically known as "two-load chopper wheel calibration" (Hiyama 1998, Mangum 2002). The cold load source will be provided by the normal interior of the FPU at about 15 K. Multiple scatterings within the FPU should all terminate at the same temperature to guarantee a good overall coupling of the radiation field to the "thermal bath" of the FPU temperature. However, the presence of the hot thermal load attached to the FPU makes it likely that small parts of the beam will be affected by radiation from the hot load when looking onto the cold load. Thus cross coupling coefficients have to be taken into account and we obtain:

$$\begin{aligned} c_{\rm cold} &= \gamma^{\rm c}_{\rm ssb}\left\{\eta_{\rm c,ssb}J_{\rm c,ssb} + (1-\eta_{\rm c,ssb})J_{\rm h,ssb}\right\} \\ &\quad + \gamma^{\rm c}_{\rm isb}\left\{\eta_{\rm c,isb}J_{\rm c,isb} + (1-\eta_{\rm c,isb})J_{\rm h,isb}\right\} + \gamma^{\rm c}_{\rm rec}J^{\rm c}_{\rm rec} + z \end{aligned} \tag{7}$$

One has to note that the system response expressed by the bandpass coefficients γ is not necessarily the same as for the astronomical observation.

The hot thermal load is a device at about 100 K attached to the FPU. When tilting the focal plane mirror it fills most of the beam, but a small part of the beam will still see the cold FPU, so that contributions from both temperatures have to be taken into account. We obtain:

$$\begin{aligned} c_{\rm hot} &= \gamma^{\rm h}_{\rm ssb}\left\{\eta_{\rm h,ssb}J_{\rm h,ssb} + (1-\eta_{\rm h,ssb})J_{\rm c,ssb}\right\} \\ &\quad + \gamma^{\rm h}_{\rm isb}\left\{\eta_{\rm h,isb}J_{\rm h,isb} + (1-\eta_{\rm h,isb})J_{\rm c,isb}\right\} + \gamma^{\rm h}_{\rm rec}J^{\rm h}_{\rm rec} + z \end{aligned} \tag{8}$$

3 Determination of calibration parameters

3.1 Basic considerations

It is obvious that it is impossible to determine all calibration quantities from these measurements when we cannot assume that the quantities stay constant when switching between the thermal loads, the sky or the reference position. The standard calibration (e.g. Kutner & Ulich 1981, Mangum 2002) assumes that all calibration quantities change only in time but not when changing the position of the focal plane mirrors.

However, all surfaces in the instrument may form low-quality Fabry-Perot interferometers leading to standing waves which modulate the transmission and the intrinsic noise of the mixer as a function of frequency. The standing waves will differ in general between the two sidebands and they change with each modification of the optical path in the instrument. Thus, the assumption of constant response functions prevents a correct treatment of standing waves.

A better treatment has to rely on a physical model of the receiver to obtain the relations between the calibration quantities in the different configurations. For SIS receivers a semi-empirical theory was provided by Tucker & Feldman (1985). This formalism was applied by Whyborn (2002) to estimate the influence of the standing waves on the intrinsic instrument noise for HIFI, with particular emphasis on standing waves towards the secondary mirror. Schieder (2002) has shown that the standing wave between the mixer and the local oscillator can have dramatic effects on the bandpass and the mixer pumping, but this path should not differ for the different measurements. No quantitative investigations have been made to study how the standing waves change the coupling to the temperature of the telescope surfaces where multiple reflections increase the contribution of ohmic losses at the reflecting surfaces.

3.2 Load calibration

3.2.1 Calibration equations

Beside these basic uncertainties we face the practical problem that additional assumptions have to be made to allow a derivation of all calibration parameters from a few measurements. Looking at the load measurements it is obvious that we can obtain only two quantities per backend channel from the two measurements.

Thus we have to assume that the standing wave pattern does not change between the two thermal loads, using the same load bandpasses γ^{l}_{ssb} and γ^{l}_{isb} and the same receiver temperature J^{l}_{rec} for both loads. As the design of the FPU was chosen to minimise standing waves towards the thermal loads it seems reasonable to assume that their difference is negligible. Because the calibration schemes proposed here use the difference in the standing wave pattern between the loads and the sky any difference in the pattern between the two loads would show up only as a second order effect. Nevertheless, this assumption may represent a limitation to any calibration scheme but there is no direct way to measure the standing waves independent of the spectral characteristics of the receiver.

As the absolute value of J^{l}_{rec} is not fixed by definition in Eqs. 7 and Eq. 8 we can define the receiver bandpass as the sum of the two load bandpasses $\gamma^{l}_{rec} = \gamma^{l}_{ssb} + \gamma^{l}_{isb}$. The zero counts z are to be measured independently by a zero termination of the backends. The coupling coefficients for the hot load $\eta_{h,ssb}$ and $\eta_{h,isb}$ and the cold load $\eta_{c,ssb}$ and $\eta_{c,isb}$ have to be measured on ground (see Roelfsema et al. 2002). Here, constants across the band can be used.

Moreover, only information on the combination of the two bandpasses can be obtained as the thermal loads provide always almost equally strong contributions in both sidebands. Thus we need additional knowledge on the sideband ratio. Here, we use a normalised sideband ratio G_{ssb} defined as $G_{ssb} = \gamma^{l}_{ssb}/\gamma^{l}_{rec}$. It does not define the ratio between both sidebands but the ratio between the response in the signal sideband and the combined response in both sidebands. Consequently, the corresponding ratio for the image sideband is $\gamma^{l}_{isb}/\gamma^{l}_{rec} = 1 - G_{ssb}$. Unfortunately, the sideband ratio can not be easily measured and it may vary when changing the LO frequency and for a given LO setting across the bandpass due to standing waves between local oscillator and mixer. Gas cell measurements as performed by Schieder (2002) promise an accuracy of 5 % basically neglecting the standing wave effects.

Although it is not necessary for a complete description of the problem, we further assume for the sake of simplicity that the coupling coefficients η_c and η_h agree in both sidebands. It is unlikely that they vary strongly between the two sidebands and it would be very difficult to measure such a variation. Then we obtain a considerable simplification of the calibration equations for the bandpass and the receiver temperature:

$$\gamma^{l}_{rec} = \frac{c_{hot} - c_{cold}}{(\eta_h + \eta_c - 1)(J_{h,eff} - J_{c,eff})} \tag{9}$$

$$\begin{aligned} J^{l}_{rec} &= \frac{\eta_h(c_{cold} - z) - (1-\eta_c)(c_{hot} - z)}{c_{hot} - c_{cold}} (J_{h,eff} - J_{c,eff}) - J_{c,eff} \\ &= \frac{(\eta_h + Y\eta_c - Y)J_{h,eff} - (\eta_h + Y\eta_c - 1)J_{c,eff}}{Y - 1} \end{aligned} \tag{10}$$

Here, the Y-factor is defined as usual by

$$Y = \frac{c_{\text{hot}} - z}{c_{\text{cold}} - z} \tag{11}$$

and the effective thermal radiation fields detectable by the receiver are given by

$$J_{\text{eff}} = G_{\text{ssb}} J_{\text{ssb}} + (1 - G_{\text{ssb}}) J_{\text{isb}} \tag{12}$$

The receiver temperature $J^{\text{l}}_{\text{rec}}$ and the average receiver bandpass $\gamma^{\text{l}}_{\text{rec}}$ determined in this way may depend arbitrarily on the spectrometer channel. In contrast to the traditional calibration equations, the imperfect coupling to the two thermal loads is explicitly taken into account here so that it provides no limitation to the calibration scheme as long as it is quantitatively known.

3.2.2 Calibration errors

To estimate the accuracy for the determination of $J^{\text{l}}_{\text{rec}}$ and $\gamma^{\text{l}}_{\text{rec}}$ one has to consider the systematic errors introduced by the assumptions discussed above and uncertainties in the parameters characterising the instrument and the statistical error from the radiometric noise during the load measurement.

3.2.2.1 Systematic calibration uncertainties

Systematic errors result from uncertainties in the knowledge of the instrumental behaviour. They are typically characterised by asymmetric error bars and result in a systematic under- or overestimate of the calibrated data. They must not be treated as random variations as proposed by Mangum (2002) because they can sum up linearly but may cancel each other as well as will be shown below. Only when distinguishing between systematic and random errors a computation of the error propagation between different quantities is possible. The computation of the systematic errors should allow to set up a budget for the accuracy in the determination of all used calibration parameters.

Systematic errors in the load calibration which can be estimated relatively straightforward come from the uncertainty of the sideband ratio G_{ssb}, the hot and the cold load temperature, and the cold and hot load coupling coefficients η_{c} and η_{h}. For the error terms we neglect the deviation of the coupling efficiencies η_{c} and η_{h} from one, considering them only as separate errors in Eqs. (9,10), because the error estimate itself does not require an accuracy better than 10 %.

The sideband ratio enters only via the effective radiation fields. Here, it is weighted by the sideband difference of the radiation field $\Delta J = J_{\text{ssb}} - J_{\text{isb}}$ relative to the constant contribution J_{eff}. From Eq. (5) we can estimate $\Delta J / J_{\text{eff}} \approx \pm 4 \nu_{\text{IF}} / \nu_{\text{LO}}$ at the hot load temperature and $\Delta J / J_{\text{eff}} \approx \pm 5 \nu_{\text{IF}} / \nu_{\text{LO}}$ at the cold load temperature. The positive upper sign applies when the signal sideband is the upper sideband, the negative sign for the opposite case. Because the calibration equations contain only the difference between the radiation fields and the radiation field from the cold load is small compared to the field from the hot load, it is only the sideband imbalance from the hot load radiation field that dominates the error. The systematic error from the uncertainty of the sideband ratio has its maximum at the edge of the bandpass, i.e. at $\nu_{\text{IF}} = 8$ GHz, and for the lowest LO frequencies.

Uncertainties in the load temperatures enter nonlinearly into the radiation field according to the Planck curve (Eq. 3). Especially the contribution from the cold load at high frequencies falls into the exponential tail resulting in sensitive reactions of the radiation field to temperature changes. Exploring the Planck curve for the HIFI frequencies gives values for $\partial J_{\rm eff}/\partial T$ of 2.0 and 1.1 at 500 GHz and of 6.2 and 1.5 at 1.9 THz for the temperatures of 15 K and 100 K, respectively. Together with the ratios between the effective radiation fields $J_{\rm c,eff}/J_{\rm h,eff}$ of $6.9\,10^{-2}$ at 500 GHz and $3.4\,10^{-3}$ at 1.9 THz we obtain reasonable upper limits for the error introduced by temperature uncertainties. Neglecting $J_{\rm c,eff}$ in the difference terms, the total systematic error in the bandpass then reads:

$$\frac{\delta\gamma^{\rm l}_{\rm rec}}{\gamma^{\rm l}_{\rm rec}} \approx -\delta\eta_{\rm h} - \delta\eta_{\rm c} \mp \frac{4\delta G_{\rm ssb}\nu_{\rm IF}}{\nu_{\rm LO}} - \frac{1.5\delta T_{\rm h} - 2.8\delta T_{\rm c}}{T_{\rm h}} \tag{13}$$

The systematic error in the receiver temperature additionally depends on the Y factor of the load measurement. However, we expect only Y-factors between 2 at 500 GHz and close to 1 at 1.9 THz so that the contribution $YJ_{\rm c,eff}$ can be neglected as well relative to $J_{\rm h,eff}$ in the error estimate. The uncertainty of the receiver temperature follows from

$$\frac{\delta J^{\rm l}_{\rm rec}}{J^{\rm l}_{\rm rec}} \approx \delta\eta_{\rm h} + Y\delta\eta_{\rm c} \pm \frac{4\delta G_{\rm ssb}\nu_{\rm IF}}{\nu_{\rm LO}} + \frac{1.5\delta T_{\rm h} - 2.8\delta T_{\rm c}}{T_{\rm h}} \tag{14}$$

To guarantee an error contribution below 1 %, the sideband ratio has to be known with an accuracy better than 15 %. The accuracy in the knowledge of the load coupling coefficients $\eta_{\rm c}$ and $\eta_{\rm h}$ enters directly as a calibration error so that an uncertainty of 0.01 would translate directly into a 1 % error contribution.

3.2.2.2 *Radiometric error*

The statistical error due to the noise in the load measurements is determined by the integration times on the thermal sources. The error in the count rates is always given by the radiometer equation:

$$\frac{\delta c}{c - z} = \frac{1}{\sqrt{\Delta\nu t_{\rm int}}} \tag{15}$$

Here, $\Delta\nu$ is the desired resolution bandwidth of the measurement and $t_{\rm int}$ is the integration time.

In contrast to the systematic errors discussed above, statistical errors are combined quadratically. For the error estimate we can neglect that $\eta_{\rm c}$ and $\eta_{\rm h}$ differ slightly from one and we obtain for the bandpass

$$\begin{aligned}\frac{\delta\gamma^{\rm l}_{\rm rec}}{\gamma^{\rm l}_{\rm rec}} &= \sqrt{\left(\frac{\delta c_{\rm hot}}{c_{\rm hot} - c_{\rm cold}}\right)^2 + \left(\frac{\delta c_{\rm cold}}{c_{\rm hot} - c_{\rm cold}}\right)^2} \\ &= \frac{1}{\sqrt{\Delta\nu t_{\rm load}}}\sqrt{\frac{(c_{\rm hot} - z)^2 + (c_{\rm cold} - z)^2}{(c_{\rm hot} - c_{\rm cold})^2}} \\ &\approx \frac{1}{\sqrt{\Delta\nu t_{\rm load}}}\sqrt{\frac{(J_{\rm h,eff} + J^{\rm l}_{\rm rec})^2 + (J_{\rm c,eff} + J^{\rm l}_{\rm rec})^2}{(J_{\rm h,eff} - J_{\rm c,eff})^2}}\end{aligned} \tag{16}$$

where the same integration time $t_{\rm int} = t_{\rm load}$ is used on the hot and the cold load.

For the receiver temperature we obtain in the same way:

$$
\begin{aligned}
\frac{\delta J^1_{\rm rec}}{J^1_{\rm rec}} &= \frac{1}{J^1_{\rm rec}(c_{\rm hot}-c_{\rm cold})^2}\sqrt{\left[(J_{\rm h,eff}+J_{\rm c,eff})(c_{\rm hot}-z)-2J_{\rm h,eff}(c_{\rm cold}-z)\right]^2\delta c^2_{\rm cold}} \\
&\qquad \overline{+\left[(J_{\rm h,eff}+J_{\rm c,eff})(c_{\rm cold}-z)-2J_{\rm c,eff}(c_{\rm hot}-z)\right]^2\delta c^2_{\rm hot}} \\
&= \frac{1}{J^1_{\rm rec}(c_{\rm hot}-c_{\rm cold})}\sqrt{(J_{\rm h,eff}-J^1_{\rm rec})^2\delta c^2_{\rm cold}+(J_{\rm c,eff}-J^1_{\rm rec})^2\delta c^2_{\rm hot}} \\
&= \frac{1}{\sqrt{\Delta\nu t_{\rm load}}}\sqrt{\frac{(J_{\rm h,eff}-J^1_{\rm rec})^2(c_{\rm cold}-z)^2+(J_{\rm c,eff}-J^1_{\rm rec})^2(c_{\rm hot}-z)^2}{J^{1\,2}_{\rm rec}(c_{\rm hot}-c_{\rm cold})^2}} \\
&\approx \frac{1}{\sqrt{\Delta\nu t_{\rm load}}}\sqrt{\frac{(J^1_{\rm rec}-J_{\rm h,eff})^2(J^1_{\rm rec}+J_{\rm c,eff})^2+(J^1_{\rm rec}-J_{\rm c,eff})^2(J^1_{\rm rec}+J_{\rm h,eff})^2}{J^{1\,2}_{\rm rec}(J_{\rm h,eff}-J_{\rm c,eff})^2}} \qquad (17)
\end{aligned}
$$

The approximate substitution of the count rates $c_{\rm hot}$ and $c_{\rm cold}$ by corresponding radiation field ratios in the last step makes use of Eqs. (7) and (8) when neglecting the imperfect coupling to the loads.

We can estimate the temperature factors from the instrument specification. We obtain the radiation temperatures $J^1_{\rm rec} \approx 84$ K, $J_{\rm h,eff} \approx 88$ K, and $J_{\rm c,eff} \approx 6$ K at 500 GHz and $J^1_{\rm rec} \approx 770$ K, $J_{\rm h,eff} \approx 61$ K, and $J_{\rm c,eff} \approx 0.2$ K at 1.9 THz. From these values the limiting error constants are

$$
\begin{aligned}
\frac{\delta\gamma^1_{\rm rec}}{\gamma^1_{\rm rec}} &\approx \frac{1}{\sqrt{\Delta\nu t_{\rm load}}}\begin{cases}2.36 & \text{at 500 GHz}\\ 18.6 & \text{at 1.9 THz}\end{cases} \\
\frac{\delta J^1_{\rm rec}}{J^1_{\rm rec}} &\approx \frac{1}{\sqrt{\Delta\nu t_{\rm load}}}\begin{cases}1.94 & \text{at 500 GHz}\\ 17.9 & \text{at 1.9 THz}\end{cases} \qquad (18)
\end{aligned}
$$

Using these values we can compute the integration times which are necessary to guarantee that the error in both quantities falls below 1 %. For a resolution bandwidth of 1 MHz, corresponding to the resolution of the wide band spectrometer (WBS), this results in integration times on each load of 0.1 s at 500 GHz and of 3.5 s at 1.9 THz. At a resolution bandwidth of 0.14 MHz, corresponding to the best resolution of the high-resolution spectrometer (HRS), we obtain integration times of 0.4 s at 500 GHz and of 25 s at 1.9 THz.

Whereas the calibration times are negligible at low frequencies, long integration times on the thermal loads have to be taken into account for an accurate bandpass calibration at high frequencies with a high spectral resolution. To increase the efficiency of these calibration measurements it might be possible to adjust the hot load temperature. Increasing the hot load temperature from 100 K to 120 K would reduce the load time by almost a factor 2 at 1.9 THz. In contrast a reduction of the hot load temperature from 100 K to 80 K would increase the load times by a factor 2.

3.3 OFF calibration

3.3.1 Basic considerations

The application of the two-load calibration to determine the bandpass of the receiver has the advantage that one case use the observation of the blank sky to reveal additional information on the system. In contrast to ground-based observations where the observation of a reference position free of emission would provide mainly information about the atmosphere, we can use such a measurement for a better characterisation of the instrument. The measurements on the blank sky can help to derive information about the difference in the standing wave pattern between the load measurements and the astronomical observation. Standing waves present in the FPU cannot be addressed in this way but as long as they are constant during all measurements they are already covered by the receiver parameters $\gamma^{l}_{\rm rec}$ and $J^{l}_{\rm rec}$ measured in the load calibration.

Unfortunately we cannot expect to obtain the difference between $\gamma_{\rm ssb}$ and $\gamma^{l}_{\rm ssb}$, $\gamma_{\rm isb}$ and $\gamma^{l}_{\rm isb}$, $\gamma_{\rm rec}$ and $\gamma^{l}_{\rm rec}$, and the forward efficiencies $\eta_{\rm l,ssb}$ and $\eta_{\rm l,isb}$ from a single measurement. Thus an additional model is needed to set up relations between these different quantities so that the information from the OFF observation can actually be used.

Until now, we have no complete model for the impact of standing waves on the different coupling coefficients. In principle they can modify the additive contribution, characterised by $\gamma_{\rm rec}$ or $J_{\rm rec}$, the receiver gain, characterised by $\gamma_{\rm ssb}$ and $\gamma_{\rm isb}$, and the telescope coupling, characterised by $\eta_{\rm l,ssb}$ and $\eta_{\rm l,isb}$ (Whyborn 2002). To keep the calibration feasible we restrict ourselves to simplified scenarios where one of the three mechanisms dominates. This results in three different standing wave calibration models.

In all approaches we will assume that the standing waves result in additive contributions $w_{\rm ssb}$ and $w_{\rm isb}$ to the corresponding coefficients. Because the OFF observation can only measure the superposition of the contributions from both sidebands an internal deconvolution is still necessary to separate both contributions. In first order the standing waves can be described by a superposition of sine waves with different periods and phases (Whyborn 2002). Thus a sideband deconvolution is possible if the number of contributing terms and their periods are not too high. However, this deconvolution is not the topic of the intensity framework outlined here so that it is to be treated elsewhere. The main difference in the standing waves between the astronomical observation and the load measurement is the path between the mixer and the subreflector. Using a length of 8.4 m for the round trip path results in a minimum standing wave period in the IF frequency domain of about 36 MHz. To resolve this standing wave ripple with Nyquist sampling a frequency resolution of 18 MHz would be sufficient. Adding another factor two as a margin we can conclude that it is sufficient to measure the standing wave effect with a frequency resolution of 10 MHz.

3.3.2 Standing waves as additive term

If the standing waves act as additive contribution to the receiver noise across the bandpass (Whyborn 2002) we can write

$$\begin{aligned}
J_{\rm rec} &= J^{l}_{\rm rec} + w_{\rm ssb} + w_{\rm isb} \\
\gamma_{\rm rec} &= \gamma^{l}_{\rm rec} \\
\gamma_{\rm ssb} &= \gamma^{l}_{\rm ssb} \\
\gamma_{\rm isb} &= \gamma^{l}_{\rm isb} \\
\eta_{\rm l,ssb} &= \eta_{\rm l,isb} = \eta_{\rm l} = {\rm const.}
\end{aligned} \tag{19}$$

In this notation we assign the standing wave term to the receiver temperature keeping the bandpasses constant. The forward efficiency is assumed to be constant across the band. All channel-to-channel variations in η_l are virtually assigned to the standing waves in $J_{\rm rec}$.

Then the sky reference equation (6) transforms into

$$c_{\rm OFF} = \gamma^{\rm l}_{\rm rec}\left[\eta_{\rm l} J_{\rm R,eff} + (1-\eta_{\rm l}) J_{\rm T,eff} + J^{\rm l}_{\rm rec} + w_{\rm ssb} + w_{\rm isb}\right] + z \tag{20}$$

The radiation field $J_{\rm R,eff}$ is the average radiation field within the whole sky beam when looking at the reference position. In principle any value can be used as long as it is well known. However, this value is hard to determine as it needs accurate knowledge about the shape of the beam including all error beams and the intensity distribution on the sky. In practice the reference position should be chosen to have emission which is very small compared to the telescope radiation across the whole IF band. Then we can use the $J_{\rm R,eff}$ contribution as error term in the measured quantities. We define the measured OFF radiation field corrected for the background intensity as

$$J_{\rm sw} = \frac{c_{\rm OFF} - z}{\gamma^{\rm l}_{\rm rec}} - J^{\rm l}_{\rm rec} - \eta_{\rm l}^{\rm guess} J_{\rm R,eff} \tag{21}$$

Here, $J_{\rm sw}$ contains the result from the OFF and the loads measurement and the correction term for a possible blank sky emission. As long as this brightness is small, any educated guess for $\eta_{\rm l}^{\rm guess}$ can be used here, e.g. 1. From Eq. (20) one can see that $J_{\rm sw}$ is attributed to the unknown standing waves and the telescope contribution:

$$(1-\eta_{\rm l}) J_{\rm T,eff} + w_{\rm ssb} + w_{\rm isb} = J_{\rm sw} \tag{22}$$

It is not possible to assign the measured $J_{\rm sw}$ clearly to any of the left hand terms in Eq. (22) as they are measured in combination. It seems, however, reasonable, to identify the average across the band as the $(1-\eta_{\rm l})J_{\rm T,eff}$ term and to assign the variation across the band to $w_{\rm ssb} + w_{\rm isb}$. The small frequency dependence of $J_{\rm T,eff}$ can be computed from Eq. (5) and is negligible as long as the sideband gain ratio of the receiver $G_{\rm ssb}$ is close to $1/2$.

3.3.3 Standing waves changing the coupling to the telescope

Another scenario is given if the standing waves do not simply increase the receiver noise but if they change mainly the coupling coefficient to the telescope radiation. This is possible when the multiple reflections in standing waves change the ohmic coupling to the temperature of the telescope surfaces. Then the list of assumptions reads as

$$\begin{aligned}
J_{\rm rec} &= J^{\rm l}_{\rm rec} \\
\gamma_{\rm rec} &= \gamma^{\rm l}_{\rm rec} \\
\gamma_{\rm ssb} &= \gamma^{\rm l}_{\rm ssb} \\
\gamma_{\rm isb} &= \gamma^{\rm l}_{\rm isb} \\
\eta_{\rm l,ssb} &= \eta_{\rm l} - w_{\rm ssb} \\
\eta_{\rm l,isb} &= \eta_{\rm l} - w_{\rm isb}
\end{aligned} \tag{23}$$

Here, the forward efficiency consists of a term $\eta_{\rm l}$ which is assumed to be a constant across the band and the two standing waves $w_{\rm ssb}$ and $w_{\rm isb}$. The negative sign is used to indicate that they increase the coupling to the telescope – not to the sky.

Then Eq. (6) reads as:

$$\begin{aligned} c_{\rm OFF} &= \gamma_{\rm rec}^{\rm l} \left[\left(\eta_{\rm l} - \frac{w_{\rm ssb}+w_{\rm isb}}{2} \right) J_{\rm R,eff} + \left(1 - \eta_{\rm l} + \frac{w_{\rm ssb}+w_{\rm isb}}{2} \right) J_{\rm T,eff} \right. \\ & \left. + \frac{w_{\rm ssb}-w_{\rm isb}}{2} \left(J_{\rm T,diff} - J_{\rm R,diff} \right) + J_{\rm rec}^{\rm l} \right] + z \end{aligned} \tag{24}$$

where we have introduced the abbreviation

$$J_{\rm diff} = G_{\rm ssb} J_{\rm ssb} - (1 - G_{\rm ssb}) J_{\rm isb} \tag{25}$$

for the effective sideband difference of the radiation fields. Treating the radiation field from the blank sky as an error term, as discussed above for the additive standing waves, we obtain:

$$\left(1 - \eta_{\rm l} + \frac{w_{\rm ssb}+w_{\rm isb}}{2} \right) J_{\rm T,eff} + \frac{w_{\rm ssb}-w_{\rm isb}}{2} J_{\rm T,diff} = J_{\rm sw} \tag{26}$$

In contrast to Eq. (22) the left hand side now contains also an expression for the sideband imbalance of the telescope radiation. Using the linear expansion of the radiation field around the LO frequency from Eq. (5) results in

$$\begin{aligned} J_{\rm T,eff} &= J_{\rm T,LO} \left[1 \pm b_{\rm T} \nu_{\rm IF} \left(2 G_{\rm ssb} - 1 \right) \right] \\ J_{\rm T,diff} &= J_{\rm T,LO} \left[2 G_{\rm ssb} - 1 \pm b_{\rm T} \nu_{\rm IF} \right] \end{aligned} \tag{27}$$

Here, the upper sign is always used if the signal sideband is the upper sideband of the receiver, ssb=usb; the lower sign applies if the lower sideband is used for the signal, ssb=lsb. Using these expressions for the standing wave terms in Eq. (26) and introducing the modified standing wave definitions $W_{\rm ssb} = w_{\rm ssb} G_{\rm ssb}$ and $W_{\rm isb} = w_{\rm isb}(1 - G_{\rm ssb})$ then leads to

$$\left(1 - \eta_{\rm l} \right) J_{\rm T,eff} + \left(1 \pm b_{\rm T} \nu_{\rm IF} \right) \left(W_{\rm ssb} + W_{\rm isb} \right) J_{\rm T,LO} = J_{\rm sw} \tag{28}$$

Like in the case of the additive standing waves, the OFF measurement is determined by two terms: the coupling to the effective telescope radiation and the standing wave contribution. Here, we find a factor which is linear in frequency in front of the standing waves modulating their amplitude across the IF band. Equivalent to the approach for the additive standing waves we can assign the measured contribution with a frequency dependence following the black-body radiation to the ordinary telescope contribution $(1 - \eta_{\rm l}) J_{\rm T,eff}$ leaving the rest of the frequency variations for the standing waves.

3.3.4 Standing waves changing the overall gain

When the standing waves modify mainly the receiver gain in both sidebands, i.e. they act mainly as a low-quality Fabry-Perot filter, we can write

$$\begin{aligned} J_{\rm rec} &= J_{\rm rec}^{\rm l} \\ \gamma_{\rm rec} &= \gamma_{\rm rec}^{\rm l} \\ \gamma_{\rm ssb} &= \gamma_{\rm ssb}^{\rm l} + w_{\rm ssb} \\ \gamma_{\rm isb} &= \gamma_{\rm isb}^{\rm l} + w_{\rm isb} \\ \eta_{\rm l,ssb} &= \eta_{\rm l,isb} = \eta_{\rm l} = \text{const.} \end{aligned} \tag{29}$$

Introducing these assumptions into Eq. (6) and using the same transformations as above for the standing waves changing the coupling to the telescope results again exactly in Eq. (28) if we interpret $W_{\rm ssb}$ and $W_{\rm isb}$ as

$$\begin{aligned} W_{\rm ssb} &= w_{\rm ssb}\frac{(1-\eta_{\rm l})}{\gamma^{\rm l}_{\rm rec}} \\ W_{\rm isb} &= w_{\rm isb}\frac{(1-\eta_{\rm l})}{\gamma^{\rm l}_{\rm rec}} \end{aligned} \qquad (30)$$

Thus the effect of a standing wave changing the coupling to the telescope and a standing wave changing the bandpass gain is indistinguishable with regard to the OFF calibration measurement. Only the resulting interpretation of $W_{\rm ssb}$ and $W_{\rm isb}$ differs and they have to be applied in a different way to calibrate the astronomical observations. Because the standing wave ripple is measured in both cases from the coupling to the warm telescope radiation, gain variations and coupling variations have the same effect on the measurement. A clear assignment of the standing waves to one of the two effects requires the observation of an additional astronomical continuum source, e.g. a planet. If the standing waves $W_{\rm ssb}$ and $W_{\rm isb}$ are strongly magnified on the continuum source relative to the blank sky they can be clearly assigned to the gain variation term; if they stay constant they are caused by a telescope coupling variation.

The distinction between these two processes and the standing waves as an additive contribution to the receiver noise could be done based on their different functional behaviour. If the amplitude of the standing waves is constant across the IF band, the resulting standing wave ripple in the OFF measurement shows as well a constant amplitude for the additive contribution but an IF frequency dependence following Eq. (28) for the gain variation. This tests relies, however, on almost ideal conditions and extremely good signal-to-noise ratios because the linear amplitude change amounts only to a few percent. Thus, the dominant standing wave process should be rather determined on ground in advance. The discrimination between the standing wave in the gain and the other two mechanisms is possible when measuring the dependence of the standing wave characteristics as a function of the temperature of a thermal source used as the signal in demonstration model level tests. The discrimination between a modification of the additive standing wave and a modification of the telescope coupling is only possible with a realistic telescope simulator which is able to reproduce the main standing wave on a surface which can be set to a temperature different from the surrounding environment.

3.3.5 Calibration errors

3.3.5.1 Systematic calibration uncertainties

The main systematic uncertainty results from the validity of the assumptions in Eqs. (20, 24, 30) but there is no direct way to quantify them. Other systematic errors are introduced by the uncertainty of the sideband ratio, the load calibration parameters, and the telescope temperature, and the "emptiness" of the blank sky.

As the calibration equations for the different standing wave mechanisms, Eqs. (22) and (28), have the same structure their accuracy can be estimated within the same frame. As a first step we have to estimate the error in the effective OFF radiation field $J_{\rm sw}$. For a shorter computation of the error estimate it is useful to substitute the calibration parameters in Eq. (21) by their definitions from Eqs. (9,10)

$$J_{\rm sw} = \frac{\eta_{\rm h}(c_{\rm OFF}-c_{\rm cold})-(1-\eta_{\rm c})(c_{\rm OFF}-c_{\rm hot})}{c_{\rm hot}-c_{\rm cold}}\left(J_{\rm h,eff}-J_{\rm c,eff}\right)+J_{\rm c,eff}-\eta_{\rm l}^{\rm guess}J_{\rm R,eff} \qquad (31)$$

The error in $J_{\rm sw}$ is affected by the blank sky radiation and the calibration parameters from the load measurement. The error contribution resulting from non-negligible emission measured on the blank sky appears as an absolute error in $J_{\rm sw}$. If the continuum contribution of $J_{\rm R,eff}$ is underestimated the coupling to the telescope will be overestimated. A line pick up from the blank sky will show up as a distortion of the derived standing wave pattern. The systematic uncertainties of the receiver temperature and the bandpass as derived in Eqs. (13) and (14) are partially amplified and partially cancelled out here because they act in Eq. (21) as a difference between two large numbers. The total error can be approximated as

$$\begin{aligned}\delta J_{\rm sw} \;\approx\;& \frac{J_{\rm T,pick}-J_{\rm c,eff}}{J_{\rm h,eff}-J_{\rm c,eff}}\delta J_{\rm h,eff}+\frac{J_{\rm h,eff}-J_{\rm T,pick}}{J_{\rm h,eff}-J_{\rm c,eff}}\delta J_{\rm c,eff}\\ &+(J_{\rm T,pick}-J_{\rm c,eff})\delta\eta_{\rm h}-(J_{\rm h,eff}-J_{\rm T,pick})\delta\eta_{\rm c}-\delta J_{\rm R,eff} \qquad (32)\end{aligned}$$

where we used the abbreviation $J_{\rm T,pick}=(1-\eta_{\rm l})J_{\rm T,eff}$ for the effective radiation pickup from the telescope structure and substituted the count rate ratios by ratios of the corresponding radiation fields as discussed in Sect. 3.2.2.

The radiation factors can be estimated for the different HIFI frequencies using the effective radiation temperatures discussed in Sect. 3.2.2. At the frequencies of 500 GHz and 1.9 THz the telescope temperature of 80 K translates into radiation temperatures of 69 K and 43 K, respectively. Assuming a forward efficiency of 98 %, we obtain an effective telescope pickup between 1.4 and 0.8 K, respectively. Then we obtain for the two limiting HIFI frequencies

$$\frac{\delta J_{\rm sw}}{J_{\rm sw}}\approx\begin{cases}-\dfrac{\delta J_{\rm R,eff}}{J_{\rm sw}}\pm 9\dfrac{\delta G_{\rm ssb}\nu_{\rm IF}}{\nu_{\rm LO}}+9\dfrac{\delta T_{\rm c}}{T_{\rm c}}-4\dfrac{\delta T_{\rm h}}{T_{\rm h}}-3.5\dfrac{\delta\eta_{\rm h}}{\eta_{\rm h}}-60\dfrac{\delta\eta_{\rm c}}{\eta_{\rm c}} & \text{at 500 GHz}\\ -\dfrac{\delta J_{\rm R,eff}}{J_{\rm sw}}\pm 4\dfrac{\delta G_{\rm ssb}\nu_{\rm IF}}{\nu_{\rm LO}}+1.5\dfrac{\delta T_{\rm c}}{T_{\rm c}}+1.1\dfrac{\delta T_{\rm h}}{T_{\rm h}}+0.7\dfrac{\delta\eta_{\rm h}}{\eta_{\rm h}}-70\dfrac{\delta\eta_{\rm c}}{\eta_{\rm c}} & \text{at 1.9 THz}\end{cases} \qquad (33)$$

The upper signs apply again when the signal sideband is the upper sideband, the lower signs in the opposite case.

The equation shows that the standing wave term reacts extremely sensitive to changes in the cold load radiation field. The coupling efficiency to the cold load is by far the dominating error term because the small $(1-\eta_{\rm c})$ contribution from the hot load measured on the cold thermal source can easily dominate the whole radiation field from the cold load. At 1.9 THz a 1 % change of the cold load coupling coefficient has about the same effect as changing the cold load temperature by 50 %. On the other hand, an uncertainty of the cold load temperature by 1 K has the same effect as an uncertainty of the hot load temperature by 9 K. Fortunately, this full systematic error does not show up in the line calibration equations in Sect. 4.3 where it is partially cancelled out again. But it is fully propagated in the determination of the forward efficiency as discussed below.

In a second step $J_{\rm sw}$ has to be split into the unknown telescope radiation contribution $J_{\rm T,pick}=(1-\eta_{\rm l})J_{\rm T,eff}$, and the standing wave contributions $w_{\rm ssb}+w_{\rm isb}$ or

$(1 \pm b_T \nu_{IF})(W_{ssb} + W_{isb})$, respectively.

$$J_{sw} = J_{T,pick} + w_{ssb} + w_{isb} \tag{34}$$

or

$$J_{sw} = J_{T,pick} + (1 \pm b_T \nu_{IF})\,(W_{ssb} + W_{isb})\,J_{T,LO} \tag{35}$$

The error of the sideband imbalance due to temperature uncertainties can be completely neglected here, because the dependence of b_T on the temperature of the telescope is extremely weak.

When assigning the frequency dependent part to the standing waves and the average to the telescope contribution, the main error comes from a possible non-zero average of the standing wave pattern. Assuming that the standing wave pattern is characterised by a superposition of sine waves, the phase of the waves may lead to a non-zero average of the standing wave across the IF band. We can estimate the maximum error from the ratio between the lowest standing wave frequency ν_w and the considered total bandwidth of the observation $\Delta\nu_{tot}$ by $w_{offset} \leq \hat{w}\nu_w/(\pi\Delta\nu_{tot})$. Here, $\hat{w}$ is the amplitude of the low frequency standing wave. With a standing wave frequency of 21 MHz and the minimum bandwidth of the HRS of 280 MHz this may lead to an error in $J_{T,pick}$ of a few percent of the standing wave amplitude. Thus the telescope coupling should rather be measured with the wide band spectrometer. Moreover, the accuracy could be further improved with a model for the fit of the standing wave terms in J_{sw}.

In a third separation step the forward scattering efficiency η_l has to be extracted from the telescope contribution. The determination of η_l relies on the knowledge of the telescope temperature. The Planck curve in the HIFI frequency range results in a derivative $\partial J_{T,eff}/\partial T_T$ of 1.2 at 500 GHz and of 1.7 at 1.9 THz. At 80 K the uncertainty in the telescope temperature translates into an uncertainty of $(1 - \eta_l)$ by

$$\frac{\delta(1-\eta_l)}{(1-\eta_l)} \approx \frac{\delta J_{sw}}{J_{sw}} - 1.2\ldots1.7\frac{\delta T_T}{T_T} \mp 4\frac{\delta G_{ssb}\nu_{IF}}{\nu_{LO}} \tag{36}$$

with the smaller factors at higher LO frequencies. The error from the sideband ratio is probably negligible relative to the error from the uncertainty of the telescope temperature. In principle Eq. (21) could also be used to derive the effective telescope temperature but it is expected that the telescope temperature can be measured much more accurately than the backward efficiency, so that the OFF calibration measurement is used to determine the latter.

3.3.5.2 *Radiometric error*

The random error in J_{sw} due to the noise in the load and the OFF measurements is determined by the integration times, so that we can derive timing constraints for a certain calibration uncertainty. For this error estimate we can neglect the blank sky radiation and the deviation of η_c and η_h from 1. Combining the different counting noise contributions in Eq. (31) provides

$$\frac{\delta J_{sw}}{J_{sw}} = \frac{J_{h,eff} - J_{c,eff}}{c_{hot} - c_{cold}} \frac{\sqrt{(c_{hot} - c_{cold})^2\delta c_{OFF}^2 + (c_{OFF} - c_{cold})^2\delta c_{hot}^2 (c_{OFF} - c_{hot})^2\delta c_{cold}^2}}{(c_{hot} - c_{OFF})^2 J_{c,eff} + (c_{OFF} - c_{cold})^2 J_{h,eff}}$$

$$\approx \frac{\sqrt{(J_{\rm h,eff}-J_{\rm c,eff})^2\delta c_{\rm OFF}^2+(J_{\rm T,pick}-J_{\rm c,eff})^2\delta c_{\rm hot}^2(J_{\rm T,pick}-J_{\rm h,eff})^2\delta c_{\rm cold}^2}}{(J_{\rm h,eff}-J_{\rm c,eff})J_{\rm T,pick}}$$

$$\approx \sqrt{\frac{1}{\Delta\nu t_{\rm OFF}}\frac{(J_{\rm rec}^1+J_{\rm T,pick})^2}{J_{\rm T,pick}^2}+\frac{1}{\Delta\nu t_{\rm load}}\frac{(J_{\rm rec}^1+J_{\rm c,eff})^2(J_{\rm T,pick}-J_{\rm h,eff})^2}{(J_{\rm h,eff}+J_{\rm c,eff})^2 J_{\rm T,pick}^2}} \quad (37)$$

The radiation factors can be estimated for the different HIFI frequencies using the effective radiation temperatures discussed above. Then we obtain for the limiting frequencies

$$\frac{\delta J_{\rm sw}}{J_{\rm sw}} \approx \begin{cases} \sqrt{\dfrac{61^2}{\Delta\nu t_{\rm OFF}}+\dfrac{68^2}{\Delta\nu t_{\rm load}}} & \text{at 500 GHz} \\ \sqrt{\dfrac{960^2}{\Delta\nu t_{\rm OFF}}+\dfrac{950^2}{\Delta\nu t_{\rm load}}} & \text{at 1.9 THz} \end{cases} \quad (38)$$

If we use the values of $\Delta\nu t_{\rm load}$ derived for a sufficiently accurate determination of the bandpass and the receiver temperature in Sect. 3.2.2 their contribution results already in a relative error of $J_{\rm sw}$ of 29 % at 500 GHz and 51 % at 1.9 THz. This error is further increased by the corresponding term from the OFF measurement having the same order of magnitude when the same integration times are used there. Thus the accurate determination of the telescope contribution and the standing waves poses much harder constraints on the integration time than the bandpass determination.

However, the determination of the telescope coupling and the standing waves does not require the same frequency resolution as the astronomical observation. Using a frequency resolution $\Delta\nu = 10$ MHz to resolve the standing waves results in an integration time on the thermal loads and the blank sky of 10 s at 500 GHz and of 1900 s at 1.9 THz for a 1 % accuracy of $J_{\rm sw}$. From these numbers it is obvious that a 1 % accuracy cannot be achieved at high frequencies. In contrast a 10 % accuracy of $J_{\rm sw}$ can be obtained within an integration time of 0.1 s at 500 GHz and 19 s at 1.9 THz. Thus, the reliable determination of the standing wave pattern requires approximately the same integration time on the thermal loads and on the blank sky as the time computed for the best HRS resolution in Sect. 3.2.2. In case of standing waves changing the instrumental gain, the random error in $J_{\rm sw}$ is propagated identically into a calibration noise of the line calibration (see Eq. 67). Thus, the timing constraints derived here may clearly result in a practical limitation of the calibration accuracy at high frequencies.

4 Differencing observations

4.1 Overview

To correct for instrumental drift effects, all astronomical observations use a differencing scheme where the astronomical source and a reference are observed in an alternating sequence. Traditionally, the difference between the counts of both measurements, $c_{\rm S} - c_{\rm R}$, is translated directly into the difference in the emission between the source and the reference position $J_{\rm S,ssb} - J_{\rm R,ssb}$. However, the coupling efficiencies, the gain, and the receiver temperature may vary systematically between the reference and the source due to changes in the optical path and the emission may also contain contributions from the image sideband. In general the full detection equation (1) of both observations has to be resolved for $J_{\mathrm{S,ssb}}$. We arrive at more complex equations containing not only differences between c_{S} and c_{R}.

HIFI is not limited to one particular reference scheme but may use four basic approaches: total power, sky chop, load chop, and frequency switch. In total power observations the whole satellite moves between the source and the reference position. Thus the optical path does not change between both signals, so that Eqs. (1) and (6) may be used with the same coupling efficiencies to compute the astronomical signal.

In sky chop observations, which we will also simply call chop observations in the following, a focal plane chopper changes the direction of the telescope beam towards a reference position up to $3'$ away from the source position. With the change of the optical path the standing wave pattern will change between both positions. This can be corrected by double beam switch observations, where the astronomical source is observed alternately in both positions of the focal plane chopper using reference positions on both sides of the astronomical source. However, it also can be corrected by explicitly measuring the standing wave pattern in both configurations using an OFF calibration as discussed in Sect. 3.3. Then the source emission $J_{\mathrm{S,ssb}}$ can be computed from Eqs. (1) and (6) by taking into account that the system response coefficients are not identical for the source and the reference. This approach can be applied as well for all sources where a double beam switch observation is not possible due to the source geometry. It requires, however, the integration time overhead of the OFF measurement.

Instead of a reference on the sky the cold load may also be used as a reference. It is obvious that the optical path then deviates completely between the source and the reference, so that two different system response functions have to be considered. The source emission will be computed from Eqs. (1) and (7) where the result from the OFF calibration enters only the equation for the source signal.

In the frequency switch reference scheme the LO frequency is shifted by a small amount so that the same position on the sky can be used for the signal and the reference but both are measured at slightly different frequencies. It is obvious that the standing wave pattern can change by the frequency switch but the general experience shows that also the parameters from the load calibration, i.e. the receiver temperature $J^{\mathrm{l}}_{\mathrm{rec}}$ and the receiver bandpass $\gamma^{\mathrm{l}}_{\mathrm{rec}}$, may differ between the two frequency settings. Thus, a complete load calibration should be performed for both frequency settings so that the astronomical observation can be calibrated from Eq. (1) using different load and OFF calibration parameters for both frequency settings.

In addition to the four different reference schemes we have to take into account the different impacts of standing waves on the calibration parameters. Corresponding to the possible dominance of one of the three mechanisms discussed in Sect. 3.3 we arrive at 12 different calibration schemes. Their properties in terms of the calibration parameters used are summarised in Table 1.

Table 1: Calibration parameters to be used in Eqs. (1,6,7) to compute the source emission for the different reference schemes and standing wave impacts. S denotes the source measurement, R the reference measurement.

Additive standing wave

		Total power	Chop	Load chop	Frequency switch
S	$\gamma_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l,S} G_{\rm ssb}$
	$\gamma_{\rm isb}$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l,S}(1-G_{\rm ssb})$
	$\eta_{\rm l,ssb}$	$\eta_{\rm l}$	$\eta_{\rm l}$	$\eta_{\rm l}$	$\eta_{\rm l}$
	$\eta_{\rm l,isb}$	$\eta_{\rm l}$	$\eta_{\rm l}$	$\eta_{\rm l}$	$\eta_{\rm l}$
	$\eta_{\rm sf,ssb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$
	$\eta_{\rm sf,isb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$
	$\gamma_{\rm rec}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l,S}$
	$J_{\rm rec}$	$J_{\rm rec}^{\rm l} + w_{\rm ssb} + w_{\rm isb}$	$J_{\rm rec}^{\rm l} + w_{\rm ssb}^{\rm S} + w_{\rm isb}^{\rm S}$	$J_{\rm rec}^{\rm l} + w_{\rm ssb} + w_{\rm isb}$	$J_{\rm rec}^{\rm l,S} + w_{\rm ssb}^{\rm S} + w_{\rm isb}^{\rm S}$
R	$\gamma_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l,R} G_{\rm ssb}$
	$\gamma_{\rm isb}$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l,R}(1-G_{\rm ssb})$
	$\eta_{\rm l,ssb}$	$\eta_{\rm l}$	$\eta_{\rm l}$	1	$\eta_{\rm l}$
	$\eta_{\rm l,isb}$	$\eta_{\rm l}$	$\eta_{\rm l}$	1	$\eta_{\rm l}$
	$\eta_{\rm sf,ssb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	1	$\eta_{\rm sf}$
	$\eta_{\rm sf,isb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	1	$\eta_{\rm sf}$
	$\gamma_{\rm rec}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l,R}$
	$J_{\rm rec}$	$J_{\rm rec}^{\rm l} + w_{\rm ssb} + w_{\rm isb}$	$J_{\rm rec}^{\rm l} + w_{\rm ssb}^{\rm R} + w_{\rm isb}^{\rm R}$	$J_{\rm rec}^{\rm l}$	$J_{\rm rec}^{\rm l,R} + w_{\rm ssb}^{\rm R} + w_{\rm isb}^{\rm R}$

Standing wave in the telescope coupling

		Total power	Chop	Load chop	Frequency switch
S	$\gamma_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l,S} G_{\rm ssb}$
	$\gamma_{\rm isb}$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l,S}(1-G_{\rm ssb})$
	$\eta_{\rm l,ssb}$	$\eta_{\rm l} - w_{\rm ssb}$	$\eta_{\rm l} - w_{\rm ssb}^{\rm S}$	$\eta_{\rm l} - w_{\rm ssb}$	$\eta_{\rm l} - w_{\rm ssb}^{\rm S}$
	$\eta_{\rm l,isb}$	$\eta_{\rm l} - w_{\rm isb}$	$\eta_{\rm l} - w_{\rm isb}^{\rm S}$	$\eta_{\rm l} - w_{\rm isb}$	$\eta_{\rm l} - w_{\rm isb}^{\rm S}$
	$\eta_{\rm sf,ssb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$
	$\eta_{\rm sf,isb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$
	$\gamma_{\rm rec}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l,S}$
	$J_{\rm rec}$	$J_{\rm rec}^{\rm l}$	$J_{\rm rec}^{\rm l}$	$J_{\rm rec}^{\rm l}$	$J_{\rm rec}^{\rm l,S}$
R	$\gamma_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l} G_{\rm ssb}$	$\gamma_{\rm rec}^{\rm l,R} G_{\rm ssb}$
	$\gamma_{\rm isb}$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l}(1-G_{\rm ssb})$	$\gamma_{\rm rec}^{\rm l,R}(1-G_{\rm ssb})$
	$\eta_{\rm l,ssb}$	$\eta_{\rm l} - w_{\rm ssb}$	$\eta_{\rm l} - w_{\rm ssb}^{\rm R}$	1	$\eta_{\rm l} - w_{\rm ssb}^{\rm R}$
	$\eta_{\rm l,isb}$	$\eta_{\rm l} - w_{\rm isb}$	$\eta_{\rm l} - w_{\rm isb}^{\rm R}$	1	$\eta_{\rm l} - w_{\rm isb}^{\rm R}$
	$\eta_{\rm sf,ssb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	1	$\eta_{\rm sf}$
	$\eta_{\rm sf,isb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	1	$\eta_{\rm sf}$
	$\gamma_{\rm rec}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l}$	$\gamma_{\rm rec}^{\rm l,R}$
	$J_{\rm rec}$	$J_{\rm rec}^{\rm l}$	$J_{\rm rec}^{\rm l}$	$J_{\rm rec}^{\rm l}$	$J_{\rm rec}^{\rm l,R}$

Table 1 - continued

Standing wave changing the gain					
		Total power	Chop	Load chop	Frequency switch
S	$\gamma_{\rm ssb}$	$\gamma^{\rm l}_{\rm rec}G_{\rm ssb}+w_{\rm ssb}$	$\gamma^{\rm l}_{\rm rec}G_{\rm ssb}+w^{\rm S}_{\rm ssb}$	$\gamma^{\rm l}_{\rm rec}G_{\rm ssb}+w_{\rm ssb}$	$\gamma^{\rm l,S}_{\rm rec}G_{\rm ssb}+w^{\rm S}_{\rm ssb}$
	$\gamma_{\rm isb}$	$\gamma^{\rm l}_{\rm rec}(1-G_{\rm ssb})$ $+\,w_{\rm isb}$	$\gamma^{\rm l}_{\rm rec}(1-G_{\rm ssb})$ $+\,w^{\rm S}_{\rm isb}$	$\gamma^{\rm l}_{\rm rec}(1-G_{\rm ssb})$ $+\,w_{\rm isb}$	$\gamma^{\rm l,S}_{\rm rec}(1-G_{\rm ssb})$ $+\,w^{\rm S}_{\rm isb}$
	$\eta_{\rm l,ssb}$	$\eta_{\rm l}$	$\eta_{\rm l}$	$\eta_{\rm l}$	$\eta_{\rm l}$
	$\eta_{\rm l,isb}$	$\eta_{\rm l}$	$\eta_{\rm l}$	$\eta_{\rm l}$	$\eta_{\rm l}$
	$\eta_{\rm sf,ssb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$
	$\eta_{\rm sf,isb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$
	$\gamma_{\rm rec}$	$\gamma^{\rm l}_{\rm rec}$	$\gamma^{\rm l}_{\rm rec}$	$\gamma^{\rm l}_{\rm rec}$	$\gamma^{\rm l,S}_{\rm rec}$
	$J_{\rm rec}$	$J^{\rm l}_{\rm rec}$	$J^{\rm l}_{\rm rec}$	$J^{\rm l}_{\rm rec}$	$J^{\rm l,S}_{\rm rec}$
R	$\gamma_{\rm ssb}$	$\gamma^{\rm l}_{\rm rec}G_{\rm ssb}+w_{\rm ssb}$	$\gamma^{\rm l}_{\rm rec}G_{\rm ssb}+w^{\rm R}_{\rm ssb}$	$\gamma^{\rm l}_{\rm rec}G_{\rm ssb}$	$\gamma^{\rm l,R}_{\rm rec}G_{\rm ssb}+w^{\rm R}_{\rm ssb}$
	$\gamma_{\rm isb}$	$\gamma^{\rm l}_{\rm rec}(1-G_{\rm ssb})$ $+\,w_{\rm isb}$	$\gamma^{\rm l}_{\rm rec}(1-G_{\rm ssb})$ $+\,w^{\rm R}_{\rm isb}$	$\gamma^{\rm l}_{\rm rec}(1-G_{\rm ssb})$	$\gamma^{\rm l,R}_{\rm rec}(1-G_{\rm ssb})$ $+\,w^{\rm R}_{\rm isb}$
	$\eta_{\rm l,ssb}$	$\eta_{\rm l}$	$\eta_{\rm l}$	1	$\eta_{\rm l}$
	$\eta_{\rm l,isb}$	$\eta_{\rm l}$	$\eta_{\rm l}$	1	$\eta_{\rm l}$
	$\eta_{\rm sf,ssb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	1	$\eta_{\rm sf}$
	$\eta_{\rm sf,isb}$	$\eta_{\rm sf}$	$\eta_{\rm sf}$	1	$\eta_{\rm sf}$
	$\gamma_{\rm rec}$	$\gamma^{\rm l}_{\rm rec}$	$\gamma^{\rm l}_{\rm rec}$	$\gamma^{\rm l}_{\rm rec}$	$\gamma^{\rm l,R}_{\rm rec}$
	$J_{\rm rec}$	$J^{\rm l}_{\rm rec}$	$J^{\rm l}_{\rm rec}$	$J^{\rm l}_{\rm rec}$	$J^{\rm l,R}_{\rm rec}$

In all cases where the table distinguishes explicitly between $\gamma^{\rm S}_{\rm rec}$, $J^{\rm S}_{\rm rec}$, $w^{\rm S}_{\rm ssb}$ and $\gamma^{\rm R}_{\rm rec}$, $J^{\rm R}_{\rm rec}$, $w^{\rm R}_{\rm ssb}$ the standing waves differ between the two settings due to a change in the optical path.

From the set of calibration parameters the sideband ratio $G_{\rm ssb}$ and the coupling coefficients for the thermal load measurements $\eta_{\rm c}$ and $\eta_{\rm h}$ have to be known from ground based tests of the instrument. The receiver bandpass and radiation temperature $\gamma^{\rm l}_{\rm rec}$ and $J^{\rm l}_{\rm rec}$ are to be determined from a load calibration at each frequency setting via Eqs. (9,10). They are independent for each backend channel. The quantities $\eta_{\rm l}$, $w_{\rm ssb}$, and $w_{\rm isb}$ have to be determined by an OFF calibration measurement using Eqs. (22) or (28). Here, an additional standing wave model is needed to separate the contributions of $w_{\rm ssb}$ and $w_{\rm isb}$. As the separation between the standing wave contributions and the forward efficiency is based on the frequency dependence, no frequency variation can be taken into account for $\eta_{\rm l}$. From the viewpoint of accuracy and efficiency the determination of $w_{\rm ssb}$ and $w_{\rm isb}$ is only reasonable at a frequency resolution of about 10 MHz and not with the full backend resolution.

The source coupling efficiency $\eta_{\rm sf}$ finally has to be determined in test observations of known astronomical objects (see Roelfsema et al. 2002).

4.2 Difference calibration equations

It makes no sense to resolve the detection equations in all the 12 cases listed above for $J_{\rm S,ssb}-J_{\rm R,ssb}$, as this is in general straight forward resulting only in a quite lengthy equation. Thus we give only a few simple examples here to demonstrate the general approach and to indicate possible obstacles and drawbacks during the calibration.

4.2.1 Total power observations with additive standing waves

The first example is the most simple situation where total power observations are used and the standing waves result mainly in an increase of the mixer noise. Then the same calibration parameters can be used for the signal and the reference measurement and subtracting them results in a complete elimination of the standing wave terms. From Eq. (1) we obtain

$$c_S - c_R = \eta_{sf}\eta_l\gamma_{rec}^l\left[G_{ssb}(J_{S,ssb} - J_{R,ssb}) + (1 - G_{ssb})(J_{S,isb} - J_{R,isb})\right] \tag{39}$$

Because both sidebands are superimposed, it is still impossible to determine $J_{S,ssb}$ from this equation, even for this most simple situation. Hence, further assumptions are necessary. The traditional approach assumes that the source radiation is restricted to the signal sideband, so that $J_{S,isb} = J_{R,isb}$. Any radiation in the image sideband is virtually assigned to the source sideband, taking into account that its calibration is incorrect then. As long as this happens for well separated lines which can be clearly assigned to one of the sidebands the use of this approach is completely justified as one can calibrate independently for the both sidebands using effectively different calibrations for lines which may be neighbouring in the backend spectrum but originating from different receiver sidebands. Then, we obtain a closed analytic formula for the astronomical calibration

$$\begin{aligned} J_{S,ssb} - J_{R,ssb} &= \frac{c_S - c_R}{\eta_{sf}\eta_l\gamma_{rec}^l G_{ssb}} && (40)\\ &= \frac{\eta_h + \eta_c - 1}{\eta_{sf}\eta_l G_{ssb}}\,\frac{c_S - c_R}{c_{hot} - c_{cold}}\left(J_{h,eff} - J_{c,eff}\right) && (41)\end{aligned}$$

which is known as the hot-cold calibration equation.

In case of continuum radiation the radiation from both sidebands can not be separated. Here, an astronomical calibration is still possible if the spectral indices of the source and the reference radiation, b_S and b_R, are known. By splitting the known continuum contribution from the lines by

$$\begin{aligned} J_{S,ssb} &= J_{S,lines} + J_{S,LO}(1 \pm b_S\nu_{IF})\\ J_{S,isb} &= J_{S,LO}(1 \mp b_S\nu_{IF})\\ J_{R,ssb} &= J_{R,lines} + J_{R,LO}(1 \pm b_R\nu_{IF})\\ J_{R,isb} &= J_{R,LO}(1 \mp b_R\nu_{IF}) \end{aligned} \tag{42}$$

we can resolve Eq. (39) for $J_{S,lines} - J_{R,lines}$.

$$\begin{aligned} J_{S,lines} - J_{R,lines} &= \frac{1}{G_{ssb}}\Bigg[\frac{c_S - c_R}{\eta_{sf}\eta_l\gamma_{rec}^l}\\ &\quad -(J_{S,LO} - J_{R,LO}) \mp (2G_{ssb} - 1)\left(J_{S,LO}b_S - J_{R,LO}b_R\right)\nu_{IF}\Bigg] && (43)\\ &= \frac{1}{G_{ssb}}\Bigg[\frac{\eta_h + \eta_c - 1}{\eta_{sf}\eta_l}\,\frac{c_S - c_R}{c_{hot} - c_{cold}}\left(J_{h,eff} - J_{c,eff}\right)\\ &\quad -(J_{S,LO} - J_{R,LO}) \mp (2G_{ssb} - 1)\left(J_{S,LO}b_S - J_{R,LO}b_R\right)\nu_{IF}\Bigg] && (44)\end{aligned}$$

The first line of Eq. (44) contains the measured total intensities. The second line represents the continuum correction characterised by the two absolute intensities $J_{\rm R,LO}$ and $J_{\rm S,LO}$ and the spectral slopes $b_{\rm R}$ and $b_{\rm S}$. In most cases the parameters of the continuum radiation from the reference position, $J_{\rm R,LO}$ and $b_{\rm R}$, are determined by large scale dust emission in the Milky Way. They can be computed from the dust density and temperature maps by Schlegel et al. (1998). If the source continuum radiation at the LO frequency is not known, Eq. (44) can be iterated to determine $J_{\rm S,LO}$ from the continuum value appearing in $J_{\rm S,lines}$. This requires, however, a clear idea about the expected line spectrum. Different approaches have to be used for spectra dominated by emission lines and spectra dominated by absorption lines. The iteration will fail in case of very rich spectra where the continuum level cannot be determined between numerous lines.

4.2.2 Total power observations with standing waves in the gain or the telescope coupling

In case of total power observations where the standing waves modify either the telescope coupling or the receiver gain, the standing wave terms $w_{\rm ssb}$ and $w_{\rm isb}$ are not cancelled out in the difference between the source and the reference observation, but they enter as modifiers to the different coupling factors. As an auxiliary step we can use a more general form of Eq. (43) by applying the split between lines and continuum already in Eq. (1)

$$\begin{aligned}
J_{\rm S,lines} - J_{\rm R,lines} &= \frac{c_{\rm S} - c_{\rm R}}{\eta_{\rm sf}\eta_{\rm l,ssb}\gamma^{\rm l}_{\rm rec}G_{\rm ssb}} \\
&\quad - \left(1 + \frac{\eta_{\rm l,isb}\gamma_{\rm isb}}{\eta_{\rm l,ssb}\gamma_{\rm ssb}}\right)(J_{\rm S,LO} - J_{\rm R,LO}) \\
&\quad \mp \left(1 - \frac{\eta_{\rm l,isb}\gamma_{\rm isb}}{\eta_{\rm l,ssb}\gamma_{\rm ssb}}\right)(J_{\rm S,LO}b_{\rm S} - J_{\rm R,LO}b_{\rm R})\,\nu_{\rm IF} \qquad (45)
\end{aligned}$$

Now, Table 1 can be used to substitute the calibration quantities. Eq. (43) is reproduced for $\eta_{\rm l,ssb} = \eta_{\rm l,isb} = \eta_{\rm l}$ and $\gamma_{\rm isb}/\gamma_{\rm ssb} = (1 - G_{\rm ssb})/G_{\rm ssb}$. For standing waves changing the gain we have to replace the term $\gamma_{\rm ssb}$ by $\gamma^{\rm l}_{\rm rec}G_{\rm ssb} + w_{\rm ssb}$ and $\gamma_{\rm isb}$ by $\gamma^{\rm l}_{\rm rec}(1 - G_{\rm ssb}) + w_{\rm ssb}$ resulting in

$$\begin{aligned}
J_{\rm S,lines} - J_{\rm R,lines} &= \frac{\eta_{\rm h} + \eta_{\rm c} - 1}{\eta_{\rm sf}\eta_{\rm l}(G_{\rm ssb} + w_{\rm ssb}/\gamma^{\rm l}_{\rm rec})}\,\frac{c_{\rm S} - c_{\rm R}}{c_{\rm hot} - c_{\rm cold}}\,(J_{\rm h,eff} - J_{\rm c,eff}) \\
&\quad - \frac{1 + (w_{\rm ssb} + w_{\rm isb})/\gamma^{\rm l}_{\rm rec}}{G_{\rm ssb} + w_{\rm ssb}/\gamma^{\rm l}_{\rm rec}}(J_{\rm S,LO} - J_{\rm R,LO}) \\
&\quad \mp \frac{2G_{\rm ssb} - 1 + (w_{\rm ssb} - w_{\rm isb})/\gamma^{\rm l}_{\rm rec}}{G_{\rm ssb} + w_{\rm ssb}/\gamma^{\rm l}_{\rm rec}}\,(J_{\rm S,LO}b_{\rm S} - J_{\rm R,LO}b_{\rm R})\,\nu_{\rm IF} \qquad (46)
\end{aligned}$$

Here, we did not replace the bandpass $\gamma^{\rm l}_{\rm rec}$ by its definition from Eq. (9) in those terms where it occurs as a denominator to the standing wave functions $w_{\rm ssb}$ and $w_{\rm isb}$ because they are measured in the OFF calibration as $w_{\rm ssb}/\gamma^{\rm l}_{\rm rec}$ and $w_{\rm isb}/\gamma^{\rm l}_{\rm rec}$ in $J_{\rm sw}$.

In case of standing waves changing the telescope coupling the forward efficiencies $\eta_{\rm l,ssb}$ and $\eta_{\rm l,isb}$ are given by $\eta_{\rm l} - w_{\rm ssb}$ and $\eta_{\rm l} - w_{\rm isb}$, respectively. The explicit calibration equation is then

$$
\begin{aligned}
J_{\mathrm{S,lines}} - J_{\mathrm{R,lines}} &= \frac{\eta_\mathrm{h} + \eta_\mathrm{c} - 1}{\eta_\mathrm{sf}(\eta_\mathrm{l} - w_\mathrm{ssb})G_\mathrm{ssb}} \frac{c_\mathrm{S} - c_\mathrm{R}}{c_\mathrm{hot} - c_\mathrm{cold}} \left(J_\mathrm{h,eff} - J_\mathrm{c,eff}\right) \\
&\quad - \frac{\eta_\mathrm{l} - w_\mathrm{isb} - G_\mathrm{ssb}(w_\mathrm{ssb} - w_\mathrm{isb})}{(\eta_\mathrm{l} - w_\mathrm{ssb})G_\mathrm{ssb}} (J_\mathrm{S,LO} - J_\mathrm{R,LO}) \\
&\quad \mp \frac{G_\mathrm{ssb}(2\eta_\mathrm{l} - w_\mathrm{ssb} - w_\mathrm{isb}) - \eta_\mathrm{l} + w_\mathrm{isb}}{(\eta_\mathrm{l} - w_\mathrm{ssb})G_\mathrm{ssb}} \left(J_\mathrm{S,LO} b_\mathrm{S} - J_\mathrm{R,LO} b_\mathrm{R}\right) \nu_\mathrm{IF} \quad (47)
\end{aligned}
$$

If continuum radiation from source and reference can be neglected the second and third term in both equations will vanish.

4.2.3 Load chop observations with additive standing waves

Load chop observations use the difference between the source signal and the signal on the cold load to correct for any temporal drift. Thus we get only information on the combined quantity $J_\mathrm{sky} = \eta_\mathrm{sf} J_\mathrm{S} + (1 - \eta_\mathrm{sf}) J_\mathrm{R}$ from this difference scheme.

In case of the additive standing waves the combination of Eqs. (1) and (7) results in

$$
\begin{aligned}
c_\mathrm{S} - c_\mathrm{cold} &= \gamma_\mathrm{rec}^\mathrm{l} \Big[(1 - \eta_\mathrm{l}) J_\mathrm{T,eff} + w_\mathrm{ssb} + w_\mathrm{isb} \\
&\qquad + G_\mathrm{ssb} \left\{ \eta_\mathrm{l} J_\mathrm{sky,ssb} - \eta_\mathrm{c} J_\mathrm{c,ssb} - (1 - \eta_\mathrm{c}) J_\mathrm{h,ssb} \right\} \\
&\qquad + (1 - G_\mathrm{ssb}) \left\{ \eta_\mathrm{l} J_\mathrm{sky,isb} - \eta_\mathrm{c} J_\mathrm{c,isb} - (1 - \eta_\mathrm{c}) J_\mathrm{h,isb} \right\} \Big] \quad (48)
\end{aligned}
$$

The first term including the telescope contribution and the standing waves is just the quantity J_sw determined in the OFF calibration. The continuum radiation from the cold load does not contain any line emission but the continuum contribution from the image sideband always has to be taken into account here. Thus the most simple calibration equation corresponds to Eq. (43):

$$
\begin{aligned}
J_\mathrm{sky,lines} &= \frac{1}{G_\mathrm{ssb}\eta_\mathrm{l}} \Bigg[\frac{c_\mathrm{S} - c_\mathrm{cold}}{\gamma_\mathrm{rec}^\mathrm{l}} - J_\mathrm{sw} \\
&\qquad - \left\{ \eta_\mathrm{l} J_\mathrm{sky,LO} - \eta_\mathrm{c} J_\mathrm{c,LO} - (1 - \eta_\mathrm{c}) J_\mathrm{h,LO} \right\} \\
&\qquad \mp (2G_\mathrm{ssb} - 1)\nu_\mathrm{IF} \left\{ \eta_\mathrm{l} J_\mathrm{sky,LO} b_\mathrm{sky} - \eta_\mathrm{c} J_\mathrm{c,LO} b_\mathrm{c} - (1 - \eta_\mathrm{c}) J_\mathrm{h,LO} b_\mathrm{h} \right\} \Bigg] \quad (49)
\end{aligned}
$$

where the known spectral coefficients for the cold load b_c and the hot load b_h from Sect. 2.1 can be used. The additive case is again the only situation where we can substitute all calibration parameters directly from the calibration measurements (Eqs. 9, 10, and 31) providing a simple closed equation for the count rates

$$
\begin{aligned}
J_\mathrm{sky,lines} &= \frac{1}{G_\mathrm{ssb}} \Bigg[\frac{\eta_\mathrm{h} + \eta_\mathrm{c} - 1}{\eta_\mathrm{l}} \frac{(c_\mathrm{S} - c_\mathrm{cold}) - (c_\mathrm{OFF} - c_\mathrm{cold})}{c_\mathrm{hot} - c_\mathrm{cold}} (J_\mathrm{h,eff} - J_\mathrm{c,eff}) \\
&\qquad + J_\mathrm{R,eff} - J_\mathrm{sky,LO} \pm (2G_\mathrm{ssb} - 1)\nu_\mathrm{IF} \left(J_\mathrm{R,LO} b_\mathrm{R} - J_\mathrm{sky,LO} b_\mathrm{sky} \right) \Bigg] \quad (50)
\end{aligned}
$$

The first difference $c_\mathrm{S} - c_\mathrm{cold}$ represents the load chop measurement on the source and the second difference $c_\mathrm{OFF} - c_\mathrm{cold}$ is the load chop difference in the OFF calibration measurement providing the telescope contributions. In the absence of drift processes, when both c_{cold} count rates agree, one can easily see that the counts on the cold load are cancelled out and we arrive formally again at equation (44) for the total power measurements with additive standing wave contributions. The continuum correction is the same in both cases because the impact of the standing waves on the baseline is completely cancelled out for additive standing waves.

4.2.4 Load chop observations with standing waves in the gain or the telescope coupling

To describe the scenario where the standing waves enter as changes in the coupling factors we have to replace Eq. (48) by the more general terminology valid for all standing wave mechanisms

$$
\begin{aligned}
c_{\text{S}} - c_{\text{cold}} = {} & \gamma_{\text{ssb}}\eta_{\text{l,ssb}}J_{\text{sky,lines}} + \left[\gamma_{\text{ssb}}\eta_{\text{l,ssb}} + \gamma_{\text{isb}}\eta_{\text{l,isb}} \pm (\gamma_{\text{ssb}}\eta_{\text{l,ssb}} - \gamma_{\text{isb}}\eta_{\text{l,isb}})b_{\text{R}}\nu_{\text{IF}}\right] J_{\text{sky,LO}} \\
& + \gamma_{\text{rec}}^{\text{l}}[J_{\text{sw}} - \eta_{\text{c}}J_{\text{c,LO}} - (1-\eta_{\text{c}})J_{\text{h,LO}} \\
& \qquad \mp (2G_{\text{ssb}} - 1)\left\{\eta_{\text{c}}J_{\text{c,LO}}b_{\text{c}} + (1-\eta_{\text{c}})J_{\text{h,LO}}b_{\text{h}}\right\}\nu_{\text{IF}}]
\end{aligned} \tag{51}
$$

Due to the lack of cancellations the standing waves enter now not only as part of J_{sw} but also via the coupling coefficients γ_{ssb} and γ_{isb} or $\eta_{\text{l,ssb}}$ and $\eta_{\text{l,isb}}$. Hence, the split between the effective telescope radiation and the standing wave contributions in Eq. (28) has to be explicitly performed.

Using the replacements for γ_{ssb}, γ_{isb}, $\eta_{\text{l,ssb}}$, and $\eta_{\text{l,isb}}$ from Table 1 we obtain the difference calibration equation for standing waves in the gain or the telescope coupling. When the standing waves enter as a modification of the gain resolving Eq. (51) for $J_{\text{sky,lines}}$ results in

$$
\begin{aligned}
J_{\text{sky,lines}} = {} & \frac{\gamma_{\text{rec}}^{\text{l}}}{(\gamma_{\text{rec}}^{\text{l}}G_{\text{ssb}} + w_{\text{ssb}})\eta_{\text{l}}}\left[\frac{c_{\text{S}} - c_{\text{cold}}}{\gamma_{\text{rec}}^{\text{l}}} - J_{\text{sw}}\right. \\
& - \eta_{\text{l}}J_{\text{sky,LO}}\left\{1 + \frac{w_{\text{ssb}} + w_{\text{isb}}}{\gamma_{\text{rec}}^{\text{l}}} \pm b_{\text{sky}}\left(2G_{\text{ssb}} - 1 + \frac{w_{\text{ssb}} - w_{\text{isb}}}{\gamma_{\text{rec}}^{\text{l}}}\right)\nu_{\text{IF}}\right\} \\
& \left. + \eta_{\text{c}}J_{\text{c,LO}} + (1-\eta_{\text{c}})J_{\text{h,LO}} \pm (2G_{\text{ssb}} - 1)\left\{\eta_{\text{c}}J_{\text{c,LO}}b_{\text{c}} + (1-\eta_{\text{c}})J_{\text{h,LO}}b_{\text{h}}\right\}\nu_{\text{IF}}\right]
\end{aligned} \tag{52}
$$

This can be simplified by substituting J_{sw} and $\gamma_{\text{rec}}^{\text{l}}$ outside of the standing wave terms

$$
\begin{aligned}
J_{\text{sky,lines}} = {} & \frac{1}{(G_{\text{ssb}} + w_{\text{ssb}}/\gamma_{\text{rec}}^{\text{l}})}\left[\frac{\eta_{\text{h}} + \eta_{\text{c}} - 1}{\eta_{\text{l}}}\,\frac{(c_{\text{S}} - c_{\text{cold}}) - (c_{\text{OFF}} - c_{\text{cold}})}{c_{\text{hot}} - c_{\text{cold}}}(J_{\text{h,eff}} - J_{\text{c,eff}})\right. \\
& - J_{\text{sky,LO}}\left(1 + \frac{w_{\text{ssb}} + w_{\text{isb}}}{\gamma_{\text{rec}}^{\text{l}}}\right) + J_{\text{R,eff}} \\
& \left. \mp \left(2G_{\text{ssb}} - 1 + \frac{w_{\text{ssb}} - w_{\text{isb}}}{\gamma_{\text{rec}}^{\text{l}}}\right)\nu_{\text{IF}}J_{\text{sky,LO}}b_{\text{sky}} \pm (2G_{\text{ssb}} - 1)\nu_{\text{IF}}J_{\text{R,LO}}b_{\text{R}}\right]
\end{aligned} \tag{53}
$$

The basic structure of the calibration equation is the same as for the additive standing waves (Eq. 50) but we find now a standing wave term in the denominator representing the modulation of the absolute line calibration due to standing waves and two additional standing wave terms in the continuum contribution (second and third line) representing the baseline distortion by standing waves.

The equivalent equation in case of standing waves entering as a modification of the telescope coupling is

$$
\begin{aligned}
J_{\mathrm{sky,lines}} &= \frac{1}{G_{\mathrm{ssb}}(\eta_{\mathrm{l}} - w_{\mathrm{ssb}})} \left[(\eta_{\mathrm{h}} + \eta_{\mathrm{c}} - 1) \frac{(c_{\mathrm{S}} - c_{\mathrm{cold}}) - (c_{\mathrm{OFF}} - c_{\mathrm{cold}})}{c_{\mathrm{hot}} - c_{\mathrm{cold}}} (J_{\mathrm{h,eff}} - J_{\mathrm{c,eff}}) \right. \\
&\quad - J_{\mathrm{sky,LO}} \left\{ \eta_{\mathrm{l}} - w_{\mathrm{isb}} - G_{\mathrm{ssb}}(w_{\mathrm{ssb}} - w_{\mathrm{isb}}) \right\} + \eta_{\mathrm{l}} J_{\mathrm{R,eff}} \\
&\quad \mp \left\{ G_{\mathrm{ssb}}(2\eta_{\mathrm{l}} - w_{\mathrm{ssb}} - w_{\mathrm{isb}}) - \eta_{\mathrm{l}} + w_{\mathrm{isb}} \right\} \nu_{\mathrm{IF}} J_{\mathrm{sky,LO}} b_{\mathrm{sky}} \\
&\quad \left. \pm (2G_{\mathrm{ssb}} - 1) \nu_{\mathrm{IF}} \eta_{\mathrm{l}} J_{\mathrm{R,LO}} b_{\mathrm{R}} \right] \qquad (54)
\end{aligned}
$$

The exact impact of the standing wave terms has another functional behaviour but we find no new effects. There is always a multiplicative modulation of the line intensities and an additive distortion of the continuum baseline.

4.2.5 Chop and frequency switch observations

For observations using the focal plane chopper to switch between two positions on the sky the standing wave pattern may vary between the two positions leading to different functions $w_{\mathrm{ssb}}^{\mathrm{S}}$, $w_{\mathrm{isb}}^{\mathrm{S}}$ and $w_{\mathrm{ssb}}^{\mathrm{R}}$, $w_{\mathrm{isb}}^{\mathrm{R}}$. For additive standing waves the difference measurement then provides

$$
\begin{aligned}
c_{\mathrm{S}} - c_{\mathrm{R}} &= \eta_{\mathrm{sf}} \eta_{\mathrm{l}} \gamma_{\mathrm{rec}}^{\mathrm{l}} \left[G_{\mathrm{ssb}}(J_{\mathrm{S,ssb}} - J_{\mathrm{R,ssb}}) + (1 - G_{\mathrm{ssb}})(J_{\mathrm{S,isb}} - J_{\mathrm{R,isb}}) \right] \\
&\quad + \gamma_{\mathrm{rec}}^{\mathrm{l}} \left[w_{\mathrm{ssb}}^{\mathrm{S}} + w_{\mathrm{isb}}^{\mathrm{S}} - w_{\mathrm{ssb}}^{\mathrm{R}} - w_{\mathrm{isb}}^{\mathrm{R}} \right] \qquad (55)
\end{aligned}
$$

When the standing waves are determined from an OFF calibration measurement the standing wave functions can be expressed by $J_{\mathrm{sw}}^{\mathrm{S}}$ and $J_{\mathrm{sw}}^{\mathrm{R}}$. Separating line and continuum terms in Eq. (55) provides an explicit calibration equation for the additive standing waves

$$
\begin{aligned}
J_{\mathrm{S,lines}} - J_{\mathrm{R,lines}} &= \frac{1}{G_{\mathrm{ssb}}} \left[\frac{1}{\eta_{\mathrm{sf}} \eta_{\mathrm{l}}} \left\{ \frac{c_{\mathrm{S}} - c_{\mathrm{R}}}{\gamma_{\mathrm{rec}}^{\mathrm{l}}} - J_{\mathrm{sw}}^{\mathrm{S}} + J_{\mathrm{sw}}^{\mathrm{R}} \right\} \right. \\
&\quad \left. - (J_{\mathrm{S,LO}} - J_{\mathrm{R,LO}}) \mp (2G_{\mathrm{ssb}} - 1) \left(J_{\mathrm{S,LO}} b_{\mathrm{S}} - J_{\mathrm{R,LO}} b_{\mathrm{R}} \right) \nu_{\mathrm{IF}} \right] \qquad (56) \\
&= \frac{1}{G_{\mathrm{ssb}}} \left[\frac{\eta_{\mathrm{h}} + \eta_{\mathrm{c}} - 1}{\eta_{\mathrm{sf}} \eta_{\mathrm{l}}} \frac{(c_{\mathrm{S}} - c_{\mathrm{R}}) - (c_{\mathrm{OFF}}^{\mathrm{S}} - c_{\mathrm{OFF}}^{\mathrm{R}})}{c_{\mathrm{hot}} - c_{\mathrm{cold}}} (J_{\mathrm{h,eff}} - J_{\mathrm{c,eff}}) \right. \\
&\quad \left. - (J_{\mathrm{S,LO}} - J_{\mathrm{R,LO}}) \mp (2G_{\mathrm{ssb}} - 1) \left(J_{\mathrm{S,LO}} b_{\mathrm{S}} - J_{\mathrm{R,LO}} b_{\mathrm{R}} \right) \nu_{\mathrm{IF}} \right] \qquad (57)
\end{aligned}
$$

We arrive again at a very simple explicit calibration equation, where the count rate difference is to be corrected only by the count rate difference measured in the OFF calibration.

In case of standing waves changing the coupling factors the same substitutions as discussed for the load chop observations in the previous section have to be performed. The resulting equations then show two different standing wave distortions to the continuum term in the second line of Eq. (57) which are subtracted from each other. The functional behaviour of these standing wave terms is identical to Eqs. (53) or (54), depending on the dominant mechanism of the standing waves. The resulting equations are somewhat more lengthy than for the load chop observations but as they contain no essential new effects we do not give the full expressions here.

The equations become even longer but do not add any new physical complexity in case of frequency switch observations. Then, we cannot longer assume that the load calibration parameters $\gamma^{\rm l}_{\rm rec}$ and $J^{\rm l}_{\rm rec}$ in the detection equation remain the same for both frequency switch phases. Consequently, different values have to be used in the difference of the count rates $c_{\rm s} - c_{\rm R}$ so that no mutual cancellations of the various quantities occur like in Eq. (55). Resolving it for $J_{\rm R,ssb} - J_{\rm R,lines}$ leads to a straight forward but extremely lengthy calibration formula. Because all essential effects are covered already by the examples above, we do not give the frequency switch equations here.

4.2.6 Summary

It is useful to compare the different explicit calibration equations discussed so far, i.e. Eqs. (44, 46, 47, 50, 53, 54, and 57). The first line in all of the above equations represents the measured overall radiation field and the subsequent lines are the continuum contributions which are subtracted to obtain a clean baseline for the lines.

It is important to notice that the continuum term contains in all cases only the radiation from the source and the blank sky in the surroundings of the source. The continuum radiation seen from the warm telescope structure cancels out in all equations by means of the OFF calibration. The continuum term always consists of a constant given by the difference between the source and the blank sky radiation field at the LO frequency and a term which is linear in IF frequency caused by a possible sideband imbalance of the radiation field or the instrumental response function.

In double-sideband receivers standing waves have always two quantitatively different effects: Except for the case of additive standing waves the line intensities are always modulated by the standing wave in their native sideband. The continuum baseline is always influenced by standing waves in both sidebands. In case of non-negligible continuum terms their modulation by standing waves leads always to a distortion of the baseline of the line spectra when the calibration does not explicitly correct for standing wave effects.

The overall radiation field term contains a kind of "standard" difference calibration formula, using the bandpass calibration from the hot-load measurement, but including possible OFF counts for a double-reference measurement. Here, it is interesting to note that there is no difference between load-chop observations and observations chopping on the sky from the viewpoint of calibration. The drawback of load-chop observations is the stronger sensitivity to instrumental drifts because the two difference terms in Eqs. (50, 53, and 54) are larger than in the sky-chop equation (57) so that smaller relative changes of the instrument may lead to larger errors in the deduced line intensities.

Instead of a single difference of count rates like in Eq. (44) the OFF calibration always provides a double difference. The double difference corrects for the standing waves in the baseline but it shows that from the viewpoint of observational noise ON and OFF measurements are equivalent. Thus, the use of an OFF calibration may be less efficient than total power measurements. Only if the system is sufficiently stable so that one OFF calibration measurement can be used for a series of source measurements this approach to the standing wave treatment is more efficient. Apart from the efficiency considerations, the OFF calibration is the only way to get a measure for the telescope pick-up and the general standing wave pattern of the instrument.

4.3 Calibration errors

Here, we will not provide a complete analysis of the error propagation for all difference schemes but discuss the general behaviour neglecting some second order effects. For the common behaviour and the differences between the calibration schemes we will consider the explicit difference calibration equations, Eqs. (44, 46, 47,50, 53, 54, and 57), and distinguish between the possible error terms in the line contribution and the continuum contributions.

4.3.1 Systematic calibration uncertainties

Causes for systematic errors are uncertainties in the source coupling efficiency $\eta_{\rm sf}$, in the telescope coupling efficiency $\eta_{\rm l}$, in the sideband ratio $G_{\rm ssb}$, and in the receiver bandpass $\gamma_{\rm rec}^{\rm l}$. An additional systematic error in the continuum contribution may arise from a wrong reference continuum value $J_{\rm R,LO}$ as well as from inaccurate spectral indices $b_{\rm S}$ and $b_{\rm R}$. Depending on the standing wave mechanism either the bandpasses $\gamma_{\rm ssb}$ and $\gamma_{\rm isb}$ or the telescope coupling $\eta_{\rm l,ssb}$ and $\eta_{\rm l,isb}$ may be influenced by standing waves. We assume here that they are only small corrections so that they provide separate error terms but can be neglected in the error terms due to uncertainties of other quantities. When we split the OFF radiation field $J_{\rm sw}$ into a constant contribution for the telescope radiation and a standing wave contribution which varies with frequency but has a zero average, all systematic errors in $J_{\rm sw}$ are assigned to $\eta_{\rm l}$ here, whereas the statistical errors from fluctuations in single channels appear mainly in the standing wave terms. Thus we can apply the systematic error of $J_{\rm sw}$ estimated in Sect. 3.3.5 as the error of $\eta_{\rm l}$.

The systematic uncertainty of the overall radiation field term, i.e. the first lines in the difference calibration equations, is

$$\frac{\delta(J_{\rm S}-J_{\rm R})}{J_{\rm S}-J_{\rm R}} \approx -\frac{\delta\eta_{\rm sf}}{\eta_{\rm sf}} - \frac{\delta G_{\rm ssb}}{G_{\rm ssb}} - \frac{\delta\eta_{\rm l}}{\eta_{\rm l}} - \frac{\delta\gamma_{\rm rec}^{\rm l}}{\gamma_{\rm rec}^{\rm l}} \tag{58}$$

Using the systematic errors of $\gamma_{\rm rec}^{\rm l}$ and $\eta_{\rm l}$ computed in Sects. 3.2.2 and Sect. 3.3.5, respectively, we obtain

$$\begin{aligned}\frac{\delta(J_{\rm S}-J_{\rm R})}{J_{\rm S}-J_{\rm R}} \;\approx\;& -\frac{\delta\eta_{\rm sf}}{\eta_{\rm sf}} - \frac{\delta G_{\rm ssb}}{G_{\rm ssb}} + \left(2-\eta_{\rm l}-\frac{J_{\rm c,eff}}{J_{\rm T,eff}}\right)\left(\delta\eta_{\rm h}+\frac{\delta J_{\rm h,eff}}{J_{\rm h,eff}-J_{\rm c,eff}}\right) \\ & + \left(2-\eta_{\rm l}-\frac{J_{\rm h,eff}}{J_{\rm T,eff}}\right)\left(\delta\eta_{\rm c}-\frac{\delta J_{\rm c,eff}}{J_{\rm h,eff}-J_{\rm c,eff}}\right) - \frac{\delta J_{\rm R,eff}+(1-\eta_{\rm l})\delta J_{\rm T,eff}}{J_{\rm T,eff}}\end{aligned} \tag{59}$$

Due to mutual cancellations not all errors from the standing wave calibration term $J_{\rm sw}$ appear with the same magnitude as total calibration errors. The cold load uncertainties are no more amplified by a large factor here.

In actual numbers at the limiting frequencies this reads as

$$\frac{\delta(J_{\rm S}-J_{\rm R})}{J_{\rm S}-J_{\rm R}} \;\approx\; -\frac{\delta\eta_{\rm sf}}{\eta_{\rm sf}} - \frac{\delta G_{\rm ssb}}{G_{\rm ssb}} \pm 4\frac{\Delta G_{\rm ssb}\nu_{\rm IF}}{\nu_{\rm LO}} - \frac{\delta J_{\rm R,eff}+0.02\delta J_{\rm T,eff}}{J_{\rm T,eff}} + \delta\eta_{\rm h}$$

$$+\begin{cases} -0.2\delta\eta_{\rm c} + \dfrac{\delta T_{\rm h}}{T_{\rm h}} + 0.04\dfrac{\delta T_{\rm c}}{T_{\rm c}} & \text{at 500 GHz} \\ -0.5\delta\eta_{\rm c} + 1.5\dfrac{\delta T_{\rm h}}{T_{\rm h}} + 0.25\dfrac{\delta T_{\rm c}}{T_{\rm c}} & \text{at 1.9 THz} \end{cases} \tag{60}$$

We see that most errors in the instrumental characteristics are transformed identically into calibration errors of the measured overall radiation field. Both the cold load coupling coefficient and the cold load temperature contribute somewhat less to the total error. The contributions from errors in the telescope temperature and in the temperature of the blank sky radiation field during the OFF calibration are strongly suppressed. In one of the first flight tests it should be checked that the standing waves $w_{\rm ssb}$ and $w_{\rm isb}$ are indeed only small corrections to the receiver temperature, bandpass or telescope coupling term so that their uncertainty has no impact on the total systematic uncertainty of the astronomical calibration.

The systematic error of the continuum contribution is determined only by the uncertainty of the sideband ratio because all uncertainties related to the source and sky coupling are covered already in the systematic error of the overall radiation field.

$$\frac{\delta(J_{\rm S,cont} - J_{\rm R,cont})}{J_{\rm S,cont} - J_{\rm R,cont}} \approx -\frac{\delta G_{\rm ssb}}{G_{\rm ssb}} \left(1 \mp 2\, \frac{J_{\rm S,LO} b_{\rm S} - J_{\rm R,LO} b_{\rm R}}{J_{\rm S,LO} - J_{\rm R,LO}} \nu_{\rm IF}\right) \tag{61}$$

In most cases the main uncertainty of the continuum contribution will stem, however, from the unknown continuum emission from the astronomical source and the sky, expressed in $J_{\rm R,LO}$, $b_{\rm S}$, and $b_{\rm R}$. A discussion of their reliability goes beyond the scope of this paper as this is rather an astrophysical than a calibration question. Because the spectral behaviour of the continuum terms is approximately linear within the whole IF band, an error in the continuum calibration will not be confused with the line spectra. It will only prevent the accurate determination of $J_{\rm S,LO}$.

4.3.2 Radiometric error

The statistical errors of the derived astronomical signal are determined by the integration times during the astronomical observation, during the load, and the OFF calibration measurements.

For the overall radiation field noise we obtain equations with growing complexity when going from total power observations with additive standing waves to chopped modes or standing waves in the coupling factors. We will show only how this increased complexity is reflected by additional noise terms. For the statistical error of the overall radiation field in total power observations with additive standing waves we obtain

$$\frac{\delta(J_{\rm S} - J_{\rm R})}{J_{\rm S} - J_{\rm R}} \approx \sqrt{\frac{\delta c_{\rm S}^2 + \delta c_{\rm R}^2}{(c_{\rm S} - c_{\rm R})^2} + \frac{\delta c_{\rm hot}^2 + \delta c_{\rm cold}^2}{(c_{\rm hot} - c_{\rm cold})^2}} \tag{62}$$

The noise in the different count rates is determined by the considered bandwidth $\Delta\nu$ and the integration time $t_{\rm int}$ corresponding to Eq. (15). Except for observations of bright planets at low frequencies, it is justified to use equal integration times on the source and the reference position in this mode. Then we can assume $c_{\rm S} - z$ and $c_{\rm R} - z$ to be quantities of the same order of magnitude. With these simplifications we obtain

$$\frac{\delta(J_\mathrm{S}-J_\mathrm{R})}{J_\mathrm{S}-J_\mathrm{R}} \approx \frac{1}{\sqrt{\Delta\nu}}\sqrt{\frac{2}{t_\mathrm{int}}\frac{{J^\mathrm{l}_\mathrm{rec}}^2}{(J_\mathrm{S,eff}-J_\mathrm{R,eff})^2}+\frac{1}{t_\mathrm{load}}\frac{(J_\mathrm{h,eff}+J^\mathrm{l}_\mathrm{rec})^2+(J_\mathrm{c,eff}+J^\mathrm{l}_\mathrm{rec})^2}{(J_\mathrm{h,eff}-J_\mathrm{c,eff})^2}} \tag{63}$$

It is important to notice the difference between the J terms on the left hand side and the J_eff terms on the right hand side. In case of line observations with contributions only in the signal sideband $J_\mathrm{eff} = G_\mathrm{ssb}J$, i.e. $J^\mathrm{l}_\mathrm{rec}/(J_\mathrm{S,eff}-J_\mathrm{R,eff}) = J^\mathrm{l}_\mathrm{rec}/[G_\mathrm{ssb}(J_\mathrm{S}-J_\mathrm{R})]$. This is often taken into account by defining a single-sideband receiver temperature as $J^\mathrm{l}_\mathrm{rec}/G_\mathrm{ssb}$, being typically a factor 2 larger than the receiver temperature $J^\mathrm{l}_\mathrm{rec}$ measured as a double-sideband temperature by the observation of continuum calibration sources here.

The first term on the left hand side is normally assumed to correspond to the signal-to-noise ratio of the observations $(S/N)_\mathrm{int} = (c_\mathrm{S}-c_\mathrm{R})/\sqrt{\delta c_\mathrm{S}^2+\delta c_\mathrm{R}^2}$ determined by the integration time. The equation shows, however, that the total noise of the observations is determined by both terms. It is nevertheless useful to compare the other noise contributions to this familiar ratio. Using Eq. (16) to substitute the second term by the statistical bandpass error results in

$$\frac{\delta(J_\mathrm{S}-J_\mathrm{R})}{J_\mathrm{S}-J_\mathrm{R}} \approx \sqrt{\frac{1}{(S/N)^2_\mathrm{int}}+\frac{{\delta\gamma^\mathrm{l}_\mathrm{rec}}^2}{{\gamma^\mathrm{l}_\mathrm{rec}}^2}} \tag{64}$$

The noise in the bandpass should be considerably smaller than $1/(S/N)^2_\mathrm{int}$ so that its contribution is small compared to the radiometric noise of the observation itself. Most HIFI observations will aim for signal-to-noise ratios between 5 and 50. The statistical bandpass accuracy $\delta\gamma^\mathrm{l}_\mathrm{rec}/\gamma^\mathrm{l}_\mathrm{rec}$ of 1 % discussed in Sect. 3.2.2 is thus sufficient for all these observations.

In case of observations using an OFF calibration to determine the standing waves the additional difference in the measured counts (see e.g. Eqs. 50 and 57) leads to a new error term equivalent to the noise from the counts on the source.

$$\frac{\delta(J_\mathrm{S}-J_\mathrm{R})}{J_\mathrm{S}-J_\mathrm{R}} \approx \sqrt{\frac{1}{(S/N)^2_\mathrm{int}}+\frac{{\delta\gamma^\mathrm{l}_\mathrm{rec}}^2}{{\gamma^\mathrm{l}_\mathrm{rec}}^2}+\frac{2}{\Delta\nu_\mathrm{sw}t_\mathrm{OFF}}\frac{{J^\mathrm{l}_\mathrm{rec}}^2}{(J_\mathrm{S,eff}-J_\mathrm{R,eff})^2}} \tag{65}$$

When the OFF calibration is performed with the same resolution bandwidth and the same integration time as the astronomical observation the total noise is thus increased by a factor $\sqrt{2}$ in spite of a total integration time (at both positions) which is longer by a factor 2. The correction of the standing wave problem by the OFF calibration would cost a factor 4 in integration time here. Fortunately, this is only a worst case estimate. In many cases one OFF calibration measurement can be used for a series of astronomical observations so that they will share the total OFF integration time. This applies to most observations mapping astronomical sources but does not help in long integrations on a single position. Independent from the share of an OFF calibration by several observations, the relative amount of the OFF calibration time can be reduced when the effective frequency resolution required to determine the standing wave pattern $\Delta\nu_\mathrm{sw}$ is lower than the resolution in the astronomical observation $\Delta\nu$. When the resolution bandwidth required to resolve the standing waves is $\Delta\nu_\mathrm{sw} \approx 10$ MHz all observations with a better frequency resolution lose only a small amount of integration time for the

OFF calibration. It can be speculated that with the development of models for the standing waves a further increase of the required resolution bandwidth may become possible. Models characterised by few parameters might be completely determined already by a measurement with an effective frequency resolution much larger than 10 MHz. Then they can be defined in a very short integration time on the thermal loads and the OFF position so that the overhead for the OFF calibration is further reduced.

Nevertheless, it has to be evaluated for each observation again whether its astrophysical scope allows to ignore possible calibration uncertainties and distortions of the baseline by standing wave ripples. In these cases the efficiency of the observation can be considerably increased by avoiding the overhead for the OFF calibration measurement.

Whenever the standing wave changes one of the multiplied coefficients, either in the bandpass or in the telescope coupling, the statistical error of the standing waves also enter the noise of the measured overall radiation field. For standing waves changing the gain of the instrument we obtain in total power measurements

$$\frac{\delta(J_{\mathrm{S}}-J_{\mathrm{R}})}{J_{\mathrm{S}}-J_{\mathrm{R}}} \approx \sqrt{\frac{1}{(S/N)_{\mathrm{int}}^2} + \frac{\delta\gamma_{\mathrm{rec}}^{1}{}^2}{\gamma_{\mathrm{rec}}^{1}{}^2} + \frac{\delta w_{\mathrm{ssb}}^2}{G_{\mathrm{ssb}}^2 \gamma_{\mathrm{rec}}^{1}{}^2}} \tag{66}$$

The noise in the standing wave can be expressed in terms of the statistical error in the effective OFF radiation field J_{sw} computed in Sect. 3.3.5. Assuming that the retrieval of the standing wave w_{ssb} from J_{sw} does not produce any additional noise, so that the noise in the telescope pickup J_{sw} is identical to the noise in the signal sideband standing wave we obtain

$$\frac{\delta(J_{\mathrm{S}}-J_{\mathrm{R}})}{J_{\mathrm{S}}-J_{\mathrm{R}}} \approx \sqrt{\frac{1}{(S/N)_{\mathrm{int}}^2} + \frac{\delta\gamma_{\mathrm{rec}}^{1}{}^2}{\gamma_{\mathrm{rec}}^{1}{}^2} + \frac{1}{G_{\mathrm{ssb}}^2}\frac{\delta J_{\mathrm{sw}}^2}{J_{\mathrm{sw}}^2}} \tag{67}$$

The corresponding equation in case of standing waves changing the telescope coupling efficiency reads

$$\frac{\delta(J_{\mathrm{S}}-J_{\mathrm{R}})}{J_{\mathrm{S}}-J_{\mathrm{R}}} \approx \sqrt{\frac{1}{(S/N)_{\mathrm{int}}^2} + \frac{\delta\gamma_{\mathrm{rec}}^{1}{}^2}{\gamma_{\mathrm{rec}}^{1}{}^2} + \frac{(1-\eta_{\mathrm{l}})^2}{\eta_{\mathrm{l}}^2 G_{\mathrm{ssb}}^2}\frac{\delta J_{\mathrm{sw}}^2}{J_{\mathrm{sw}}^2}} \tag{68}$$

In case of chopped/frequency switch observations using the OFF calibration to correct for standing waves in the baseline, the OFF noise corresponding to Eq. (65) has to be added in the square root.

It is important to notice the essential difference in the standing wave noise term between Eqs. (67) and (68). In case of standing waves changing the telescope coupling the OFF calibration noise is suppressed by the factor $(1-\eta_{\mathrm{l}})/\eta_{\mathrm{l}}$, i.e. ≈ 0.02. In case of standing waves changing the gain of the instrument the full OFF calibration noise enters the radiation field calibration.

Thus the calibration of the overall radiation field for the standing waves in the gain sets much harder constraints on the OFF integration times than the calibration in any of the other scenarios. Here, we can compare the integration time requirements for the standing wave measurement from Eq. (38) with the requirements for the baseline calibration resulting from Eq. (65). In case of strong line signals this may result in the need for an integration time on the OFF position exceeding the estimate discussed above for the chopped observations. As shown in Sect. 3.3.5 a 1 % accuracy can only be achieved at low frequencies within reasonable integration times. At high frequencies noise contributions up to 10 % are unavoidable. In some cases it might thus be better to ignore the impact of the standing wave term in the absolute calibration factor for the overall radiation field using it only in the additive terms. This means that in Eqs. (46) and (53) the factor $(G_{\rm ssb} + w_{\rm ssb}/\gamma^1_{\rm rec})$ is replaced by $G_{\rm ssb}$. From the current knowledge on the standing waves it is not clear whether one introduces a bigger error into the line calibration by ignoring the standing wave term in the instrumental gain or by using the standing wave correction in this term with an error of up to 10 %. One has to keep in mind, however, that even when omitting the standing wave term in the gain the OFF calibration should not be completely dropped. It is still essential for an effective removal of the standing wave ripples from the baseline providing the continuum level.

The statistical error of the continuum contribution is always dominated by the noise in the standing wave measurement. We obtain

$$\frac{\delta(J_{\rm S,cont} - J_{\rm R,cont})}{J_{\rm S,cont} - J_{\rm R,cont}} \approx \frac{\delta J_{\rm sw}}{J_{\rm sw}} \tag{69}$$

for standing waves in the gain and in the telescope coupling. Because the continuum can be averaged over the the whole IF band the noise in $J_{\rm sw}$ can be computed for a very large effective noise bandwidth so that this error is always small.

Altogether, the noise considerations have shown that there is a price to be paid for the accurate determination of the standing wave pattern. Because the standing wave is measured from the weak telescope radiation pickup, long integration times are required for the OFF calibration. The OFF integration time can always be shorter than the source integration time by the ratio between the frequency resolution of the actual observations and the resolution required to measure the standing wave period. In case of stable standing waves one OFF measurement can also be used for the calibration of several astronomical observations. From the requirements on the integration times it turns out, however, that in case of standing waves modifying the instrumental gain, a correction of their impact on the line strengths is not always possible with a high accuracy. In all observing modes the OFF calibration is still essential for an accurate determination of the spectral baselines.

5 Nonlinear response

In principle it is not guaranteed that the instrumental response to any of the input radiation fields is linear as assumed in Eq. (1). Only the receiver temperature is an additive constant by definition. The combination of the receiver, the IF amplifiers and the backends may show some nonlinear response. Deviations from a linear behaviour are expected mainly in the IF branch including the spectrometers (e.g. Siebertz 2002). However, because the detection mechanism does distinguish between photons of different origin the response functions to contributions from $J_{\rm S}$, $J_{\rm R}$, and $J_{\rm T}$ have to agree. Thus we may describe any nonlinearity as a modification of the linear count rates

$$c_{\rm S,measured} = f(c_{\rm S}) \tag{70}$$

where the count rate on the right hand side represents any of the count rates discussed in the previous sections.

Practically, we don't expect strong deviations of f from a linear behaviour so that the determination of a few expansion parameter should be sufficient. The functional behaviour should be determined in tests with well defined sources on ground. To obtain the function f one has to scan the possible range of c_S values measuring $c_{S,\mathrm{measured}}$. Within a limited range, the nonlinearity in the IF branch can be determined when the receiver is used as noise source at a bias point outside of the superconducting gap. To include measurements on the nonlinearity of the receiver response one can adjust the hot load temperature to scan a range of count rates. However, it is difficult to achieve a high accuracy in this way as the temperature determination itself is subject to several uncertainties (Roelfsema et al. 2002).

If the nonlinearity function is known each intensity calibration has to start with a transformation of the measured counts $c_{S,\mathrm{measured}}$ by f^{-1} onto the linear scale of c_S. Then the differencing schemes from Sect. 4 can be applied or the calibration parameters can be determined using the equations from sections 3.2 and 3.3.

6 Summary

We propose a new calibration scheme for the planning and reduction of HIFI observations taking explicitly into account **i)** that the large intermediate frequency of HIFI requires a separate treatment of both sidebands, **ii)** that the problem of standing waves between the subreflector and the receiver can be solved by means of a separate OFF calibration measurement, **iii)** that the telescope temperature is less accurately known than on ground-based telescopes, and **iv)** that atmospheric issues can be dropped for HIFI.

In the new calibration scheme each observation is characterised by 7 intensity calibration parameters: the bandpass in the source sideband γ_{ssb} and in the image sideband γ_{isb}, the forward efficiencies in both sidebands $\eta_{\mathrm{l,ssb}}$ and $\eta_{\mathrm{l,isb}}$, the source coupling efficiencies $\eta_{\mathrm{sf,ssb}}$ and $\eta_{\mathrm{sf,isb}}$, and the receiver noise given by $\gamma_{\mathrm{rec}} J_{\mathrm{rec}}$. Depending on the dominant process causing standing waves they are related by different equations to 9 basic measurable calibration quantities. These are the total receiver bandpass $\gamma_{\mathrm{rec}}^{\mathrm{l}}$ and the receiver temperature $J_{\mathrm{rec}}^{\mathrm{l}}$, determined in the load calibration measurement, the sideband ratio G_{ssb} and the coupling coefficients to the cold and the hot load η_{c} and η_{h}, determined on ground, the source coupling efficiency η_{sf}, determined from beam measurements, and the effective forward efficiency η_{l} and the standing waves w_{ssb} and w_{isb}, determined in the OFF calibration measurement. All quantities except the forward efficiency η_{l} might be different for each backend channel but practical reasons show that no frequency dependence can be determined for the hot load coupling efficiency η_{h}, the sideband ratio G_{ssb}, and the source efficiency η_{sf}, so that these are treated channel independent.

The correction for standing wave effects is based on the availability of two thermal loads in HIFI. They allow to use the measurement on the blank sky (OFF calibration) to derive the properties of the standing wave difference between a load measurement and a sky measurement. The OFF calibration provides only information on the superposition of the standing waves in both sidebands, w_{ssb} and w_{isb}, so that a parametrised model is required to deconvolve them. Such a model is available from the analysis of the optical design of the instrument (Whyborn 2002).

In the double-side band design of HIFI the standing waves have always two effects:

they modulate the continuum level providing distortions to the spectral baseline of the signal and they modulate the absolute calibration of the lines. Both effects have a different functional behaviour and their correction imposes different constraints on the time spent for the OFF calibration. The standing wave ripple in the continuum baseline can be suppressed when the observation of the source and the reference uses the same optical path, i.e. in total power observations. The modulation of the line signal is only negligible when the standing waves do not change the different coupling factors in the instrument, especially the gain, but modulate only the receiver noise. Unfortunately, the dominant effect of the standing waves is not yet known.

At high frequencies, where the receiver noise is very large compared to the telescope pickup and the dominant period of the standing waves corresponds to a velocity difference of only 4 km/s, the standing wave correction imposes severe constraints on the integration times required for the load calibration and the OFF measurement. The OFF calibration may require a relatively large part of the total observing time. The total efficiency of an observation including the standing wave correction grows almost proportional to the stability of the standing waves in the instrument. It is also enhanced when the frequency of the standing wave signature allows to smooth the OFF data over a broader bandwidth than the astronomical observations.

If the OFF calibration is used it allows to correct for all standing wave effects in the path between the focal plane unit and the primary mirror. It measures the standing wave difference between two phases of chopped observations providing a clean way to remove all standing wave ripples from the spectral baseline of the observations and to include the effect of the modulation of the line strengths in their calibration. Thus it enables the use of sky-chop, load-chop or frequency switch observations even for sources with broad lines or a considerable continuum contribution. A reasonable use of the standing-wave calibration is, however, only possible if the design of the satellite is optimised to provide a stability of the standing wave pattern over time scales which are considerably longer than the stability time of the instrument itself. Moreover, it relies on the knowledge of the dominant coupling mechanism between the standing waves and the instrument. That makes it absolutely necessary that corresponding tests as proposed in Sect. 3.3.4 are performed before launch.

We have also provided an accurate estimate of the error budget of the calibration showing that a high accuracy of the calibration is only possible when the sideband ratio $G_{\rm ssb}$ and the coupling coefficients to the hot and the cold load, $\eta_{\rm h}$ and $\eta_{\rm c}$, respectively, are very well known. From the computation of the systematic errors it is possible to put clear limits on the accuracy of the parameters entering the intensity calibration required to achieve a particular calibration accuracy.

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A Summary of calibration quantities

Symbol	Explanation[1,2]	Sect.	Guess
γ_{ssb}	signal bandpass in the signal sideband	2.1	
γ_{isb}	signal bandpass in the image sideband	2.1	
G_{ssb}	signal sideband gain	2.1	
$1 - G_{\mathrm{ssb}}$	image sideband gain	2.1	
$\eta_{\mathrm{l,ssb}}$	forward efficiency in the signal sideband	2.1	≈ 0.98
$\eta_{\mathrm{l,isb}}$	forward efficiency in the image sideband	2.1	≈ 0.98
$(1 - \eta_{\mathrm{l,ssb}})$	backward efficiency (part of the beam terminating on the warm telescope structure) in the signal sideband	2.1	≈ 0.02
$(1 - \eta_{\mathrm{l,isb}})$	backward efficiency in the image sideband	2.1	≈ 0.02
$\eta_{\mathrm{sf,ssb}}$	source efficiency in the signal sideband	2.1	
$\eta_{\mathrm{sf,isb}}$	source efficiency in the image sideband	2.1	$= \eta_{\mathrm{sf,ssb}}$
$\eta_{\mathrm{c,ssb}}$	cold load coupling coefficient in the signal sideband	2.3	≈ 0.996
$\eta_{\mathrm{c,isb}}$	cold load coupling coefficient in the image sideband	2.3	$= \eta_{\mathrm{c,ssb}}$
$\eta_{\mathrm{h,ssb}}$	hot load coupling coefficient in the signal sideband	2.3	≈ 0.99
$\eta_{\mathrm{h,isb}}$	hot load coupling coefficient in the image sideband	2.3	$= \eta_{\mathrm{h,ssb}}$
γ_{rec}	receiver bandpass	2.1	$\gamma_{\mathrm{ssb}} + \gamma_{\mathrm{isb}}$
J_{rec}	receiver temperature	2.1	see Spec.
z	zero counts of the backend	2.1	
J_{S}	radiation intensities from the source	2.1	
J_{R}	radiation intensities from the sky outside of the source	2.1	
J_{T}	radiation intensities from the sum of the telescope contributions within the beam	2.1	$B_\nu(80\ \mathrm{K})$
J_{h}	radiation intensities from the hot thermal load	2.3	$B_\nu(100\ \mathrm{K})$
J_{c}	radiation intensities from the cold thermal load	2.3	$B_\nu(15\ \mathrm{K})$
c_{ON}	spectrometer count rate on the astronomical source position	2.1	
c_{OFF}	spectrometer count rate on the blank sky	2.2	
c_{cold}	spectrometer count rate on the thermal cold load	2.3	
c_{hot}	spectrometer count rate on the thermal hot load	2.3	
b	linear coefficient of series expansion of the continuum radiation field J around the LO frequency	2.1	
Y	Y-factor from a hot and cold thermal load measurement	3.2.1	
J_{eff}	effective radiation intensity from both sidebands at a given sideband ratio G_{ssb}	3.2.1	
J_{diff}	difference in the radiation intensity between both sidebands at a given sideband ratio G_{ssb}	3.3.3	
J_{sw}	telescope and standing wave intensity	3.3.5	
$J_{\mathrm{T,pick}}$	effective radiation intensity in both sidebands picked up within the telescope beam	3.3.5	
w_{ssb}	standing wave contribution in the signal sideband	3.3.2	
w_{isb}	standing wave contribution in the image sideband	3.3.2	

[1] The quantities γ_{ssb}, γ_{isb}, G_{ssb}, $\eta_{\mathrm{l,ssb}}$, $\eta_{\mathrm{l,isb}}$, γ_{rec}, J_{rec} depend on the exact optical path of the measure-

ment so that they are not unique numbers but may differ between the different settings of the focal plane chopper mirror, i.e. different chop positions on the sky and the hot and load thermal loads.

[2] The role of the standing wave terms w_{ssb} and w_{isb} depends on the process providing the dominant influence on the observation as discussed in Sect. 3.3.1.

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