Bandslope Effects On Sensitivity In Interferometers With Digital Correlators

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15 January 2002

Abstract: In an interferometer with a digital correlator, sensitivity will be reduced by receiver and IF bandpass slopes. Numerical calculations have been carried out to estimate these losses for developing a flatness specification for the ALMA receivers and IF system. For the 3-level digitizer studied here a 2-dB slope across the band gives an efficiency reduction of about 4 % in the frequency channels where the gain is least. There is also an increase of efficiency of a similar magnitude in the high-gain channels. For continuum observations the channels are weighted according to the channel signal-to-noise and averaged, resulting in an efficiency reduction of ~1 %. From these considerations it should be acceptable to have a gain slope of ~2 dB across the digitizer input bandwidth. Since the ALMA digitizer will have a higher number of bits it may be possible to go to 3 dB across the band.

1. Introduction

Specifications for the acceptable gain slope through the IF of the ALMA receivers should take in to account any effects on sensitivity. Thompson [1] has evaluated the effect of non-uniform gain in the receiving chain in an interferometer. In the simplest case a continuum correlator (multiplier) was considered. A slope in gain was shown to result in a very small reduction in sensitivity. It is exactly equivalent to the sensitivity reduction that would be expected for a single dish with the correlator replaced by a square-law detector. In such cases the degradation is often quantified by the 'effective bandwidth' (Tiuri in [2]). When a correlator with several spectral channels is used this reduction in sensitivity may be avoided by averaging the channels together with the appropriate weighting, provided the slope across each channel is sufficiently small. When the correlator is fed by a digitizer with very few sample levels there is an additional loss of sensitivity. Digitization noise is spread throughout the band in a fairly uniform distribution. The spectral channels where the input gain is lower are therefore expected to have a lower signal-to-noise ratio (SNR). Some simple numerical results derived from simulations are presented below.

2. Numerical Study

Fig. 1 shows a schematic of the system being analyzed. A small coherent noise signal, v_s, is applied to two transmission chains. Each of these adds some noise voltage v_1, v_2, (representing all the uncorrelated noise sources) and the two voltages $v_s + v_1$ and $v_s + v_2$ are digitized and cross-correlated. A gain slope is applied to the noise before injection into the digitizers, and the digitizer threshold levels are adjusted to the optimum rms value relative to the digitizer levels [3]. The slope is linear in dB and specified as the total change across the band. The particular correlator architecture is not important and the results will be the same for a lag-correlator and an FX (Fourier transform and multiply) correlator. For the present study the digitizer outputs were first Fourier transformed and then multiplied to give the correlated output. A more detailed description of the analysis method may be found in the Appendix.

To estimate the impact of digitization, a simple two-bit three-level quantization scheme was modeled. This produces 0 or ±1 depending on the input voltage relative to two threshold levels, which are symmetric about zero volts. With the correct choice of thresholds this gives a factor of 0.81 reduction in sensitivity relative to a perfect analog correlator [3]. There are other quantization schemes that have higher efficiency, but the essential features of the problem are illustrated with this example.

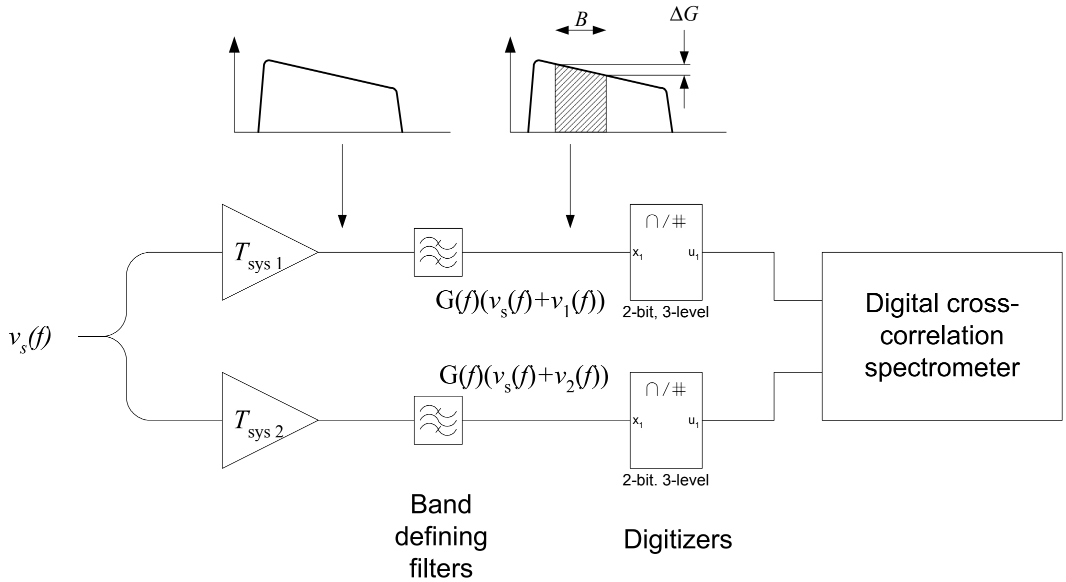

Fig. 1. A typical digital cross-correlator scheme. A correlated signal is sent into two paths with independent noise sources. The actual architecture of the digital part of the correlator is unimportant.

Fig. 2 presents the outcomes of the simulations for 2-dB steps from zero to 10 dB. The flat-bandpass case agrees perfectly with the theoretical efficiency factor of 0.81. When a gain slope is applied there is a reduction of SNR at the low-gain end of the band, but an increase at the high-gain end. This increase is at first sight unexpected, but an extreme case of a non-flat band is one that is uniform for half the bandwidth and zero for the other half. It can be seen that this is equivalent to oversampling the narrow band, and indeed the numerical results for this case conform to the theoretical efficiency factor of 0.88.

For spectral line data the sensitivity loss may be found directly from these results. For continuum measurements the channels must be added together, appropriately weighted for the noise in the individual channels. Assuming the channels are narrow (*i.e.*, the gain is essentially constant across any individual channel), this can be approximated by an integral of the curves in Fig. 2, weighted by the inverse-square of the SNR. The results of this are shown in Fig. 3

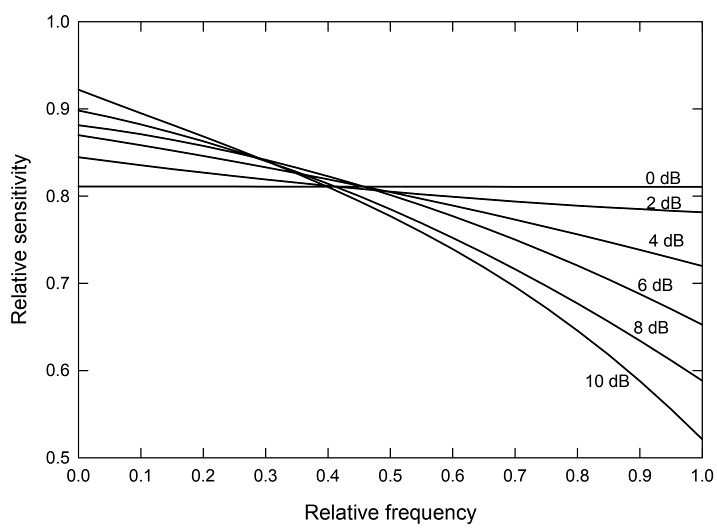

Fig. 2. Sensitivity of a 3-level correlator relative to a perfect analog correlator for various slopes in gain across the input bandwidth.

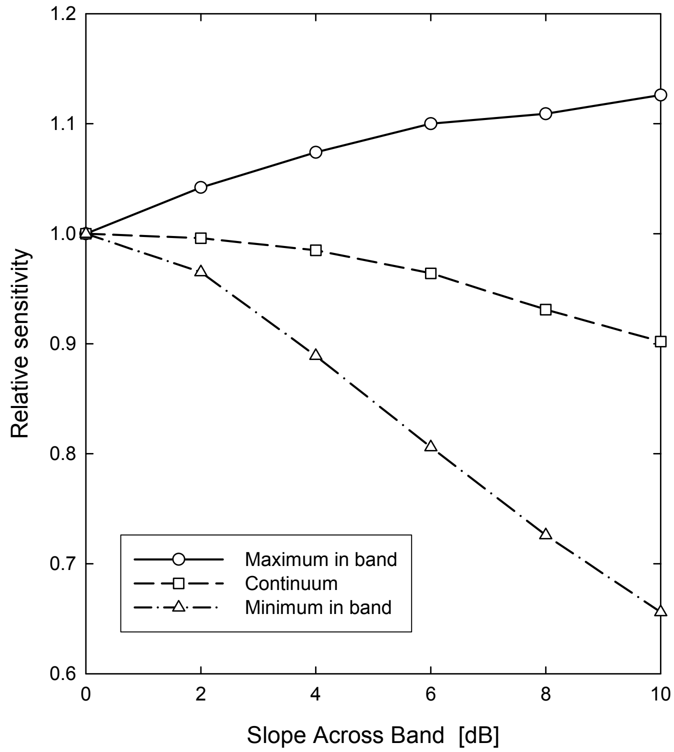

Fig. 3. Effective continuum sensitivity for a 3-level digitizer with various gain slopes across the band. The "maximum" and "minimum" correspond to the best and worst edges of the bands in the spectral mode, and the "continuum" corresponds to an optimally weighted sum of the channel measurements.

3. Conclusions

The effect of digitization in the presence of a slope in the input bandpass gain is to reduce the interferometer efficiency where the gain is low and increase it where it is high. This will adversely affect spectroscopic observations where the lines fall in the low-gain parts of the band. In continuum measurements, using the optimum noise weighting gives negligible degradation of efficiency for gain slopes less than ~2 dB. For ALMA a finer digitization scheme will be used so the specification for bandpass slope can be relaxed relative to these calculations. Probably a slope of at least 3 dB across the input to each digitizer sub-band will be acceptable.

4. Acknowledgements

I would like to thank Dick Thompson and Dave Woody for useful discussions on this topic.

5. Appendix: Details of Calculation

All of the simulations were carried out in Mathcad 2001 [4]. Although the implementation as described below is not a true representation of the actual flow of data in a digital correlator, it is mathematically consistent and in particular it reproduces the statistical results.

Each simulation follows the following steps.

1. Create a real sequence of 2^{20} numbers $v_s(f_i)$ with a Gaussian distribution having an rms of $<v^2_s>^{1/2}$ to represent the signal spectrum. The frequencies f_i are uniformly spaced over the input bandwidth, B.

2. Create two complex sequences of 2^{20} numbers with Gaussian distributions $v_1(f_i)$ and $v_1(f_i)$ with rms values of $<v^2{}_1>^{1/2}$ and $<v^2{}_2>^{1/2}$ to represent the spectra of noise from the atmosphere, antennas, receivers, *etc*.
3. Add the signal to the two noise spectra to get the nominal input spectra for the cross-correlator, $v_a = v_s + v_1$ and $v_b = v_s + v_2$.
4. Weight each of the points to represent a gain slope that is linear in dB across the band.
5. Fourier transform the two spectra $v_a(f)$ and $v_b(f)$ to time series $V_a(t)$ and $V_b(t)$.
6. Digitize the two time sequences to 0 or ±1, with the threshold levels set at $\pm 0.65 \times \mathrm{rms}(V_a)$ and $\pm 0.65 \times \mathrm{rms}(V_a)$ to give the discrete versions of the time sampled streams V'_a and V'_b respectively.
7. Inverse Fourier transform these sequences to get the digitized estimates of the spectra, v'_a and v'_b.
8. Obtain the cross-correlations for the digitized and undigitized spectra, $s = \mathrm{v}_a \mathrm{v}_b$ and $s' = v'_a v'_b$.
9. Divide the cross-correlations into 20 frequency bins. The SNR for each bin is computed as the average over the rms, giving the 'input' value, *SNR* from *s*, and the 'output' value, *SNR*' from *s*'. (This process is essentially equivalent to producing lower resolution spectra from consecutive time segments and averaging the spectra together.)
10. Compute the ratio *SNR*/*SNR*' to represent the sensitivity degradation in each frequency channel.

This series of steps is repeated about twenty times for each value of the gain slope to improve the estimate. To estimate the continuum sensitivity the data from the above simulations are fitted with polynomials and integrated in Mathcad with a weighting inversely proportional to the square of the SNR.

6. References

[1] A. R. Thompson, "Degradation of sensitivity resulting from bandpass slope," NRAO, ALMA Memo Series No. 388, Aug. 2001.

[2] J. D. Kraus, *Radio Astronomy*, 2nd ed. Powell, OH: Cygnus-Quasar Books, 1986.

[3] A. R. Thompson, J. M. Moran, and G. W. Swenson Jr, *Interferometry and Synthesis in Radio Astronomy*. New York: Wiley, 1986.

[4] "Mathcad 2001." Cambridge: MathSoft Engineering & Education, Inc.