ALMA Memo 331

On-The-Fly Fringe Tracking

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2000-Oct-27

Abstract

Fringe tracking methods for on-the-fly interferometry are considered. Several options are discussed, and it is concluded that those in which the desired phase function is discontinuous are undesirable. If the phase center is left at a fixed position on the sky while the antenna beams are scanned, then some loss of coherence occurs when the beams are away from the fixed position. The size of this loss is calculated, and it is found to be proportional to the total scan time and independent of observing frequency. For a given maximum loss and thus a maximum scan time, the number of beamwidths covered depends on the scanning speed.

For the worst-case ALMA parameters and 2% maximum loss, a linear scan can last about 6 sec and can cover 18 arcmin (limited by antenna speed) or 379 beamwidths (limited by correlator dumping rate), whichever is less.

Introduction

Fringe tracking for the ALMA telescope was discussed in [1], along with some related issues. However, the case of on-the-fly (OTF) interferometry has not been explicitly considered. In this technique, all antenna beams of the array (or subarray) are continuously scanned together across a source that is larger than the single-dish beam, while the resulting signals are cross-correlated using a series of short integrations. If several integrations occur per beamwidth of antenna motion, then the source visibility is well sampled. An image of the extended source can later be computed by the "mosaicing" technique [2]. This is considerably more efficient in observing time than the alternative of observing at discrete pointings, since the overhead of starting and stopping the antenna motion is avoided.

It is not obvious how to accomplish the necessary phase and delay tracking during an OTF observation. Three approaches have been proposed:

a. Track a fixed point on the sky during each integration, typically the point to which the antenna beams point at the middle of that integrating time, and switch to another fixed point during each successive integration.

b. Track the center of the antenna beams, which means that the phase/delay center on the sky moves continuously with the beams.

c. Track a fixed point on the sky for the full duration of an OTF scan, and switch to a new phase/delay center only between scans, when the antenna is off source and no integration is occurring.

Option *a* would allow the simplest data analysis, but it is the most difficult to implement. Option *c* is the easiest to implement. Other advantages and disadvantages are discussed in the next section, and the consequences of choosing *c* are analyzed in the following section.

Design Considerations

Tracking a different point for each integration requires that the phase change discontinuously (or nearly so), and that the changes be synchronized with the end/beginning of a correlator integration. Both of these things are technically difficult. The direct digital synthesizers (DDSs) can be programmed to produce nearly instantaneous phase changes, but the phase response of the LO is filtered by that of the phase locked loop. Synchronization with the correlator requires that the full propagation delay from the antenna to the correlator proper (including fixed and variable parts) be taken into account.

Continuous tracking of the beam center is possible, but it results in a smearing of the visibility function during the integrating time and may be difficult to account for in the imaging process.

Tracking a fixed point on the sky throughout the OTF scan is easy. But at the ends of the scan, when the beam is offset from the phase center, a loss of sensitivity occurs because the fringe frequency is not correct. The loss increases with the product of integrating time τ and the number of beamwidths of offset N. For a linear scan across the source, $N\tau$ is proportional to the total scanning time t ($t = 4N\tau$ for a scan width of $\pm N$ beamwidths from the center and two integrations per beamwidth). For more complicated scanning patterns, like spirals, the total time may be different.

Coherence Loss for a Linear Scan with Fixed Phase Center

From [3], p. 91, eq. 4.23, the fringe frequency is given by

$$f = -\omega[(X/\lambda)\cos\delta\sin H + (Y/\lambda)\cos\delta\cos H]$$

where X and Y are the E-W and N-S baseline components, respectively; λ is the observing wavelength; δ is the declination; H is the hour angle, and $\omega = 2\pi/(24\times 3600) = 7.27\times 10^{-5}\,\mathrm{rad/sec}$ is the angular rotation rate of the earth. Taking the partial derivatives w.r.t. δ and H gives

$$df/d\delta = -\omega[-(X/\lambda)\sin\delta\sin H - (Y/\lambda)\sin\delta\cos H]$$

and

$$df/dH = -\omega[(X/\lambda)\cos\delta\cos H - (Y/\lambda)\cos\delta\sin H].$$

In each case, the maximum value is $\omega X/\lambda$ or $\omega Y/\lambda$. We thus take the maximum rate of change of fringe frequency to be $\omega D/\lambda$, where D is the maximum baseline length of the array.

The full width to half-power of the antenna beam is approximately $1.2\lambda/d$, where d the diameter of the antenna. At an offset of N beamwidths, the maximum fringe frequency change is therefore approximately

$$\begin{aligned}\Delta f &= (\omega D/\lambda)(1.2N\lambda/d)\\ &= 1.2N\omega D/d.\end{aligned}$$

Note that this is independent of wavelength. The longer the wavelength, the slower the fringes, but the larger the primary beamwidth by the same factor*.

For small offsets and short integrations, the phase error varies nearly linearly at rate Δf. The value at the center of the integration can be computed and removed in post processing, but the variation causes the coherence to be reduced by a factor

$$\begin{aligned}C &= \frac{1}{2\pi\tau\Delta f}\int_{-\pi\tau\Delta f}^{\pi\tau\Delta f}\cos\phi\, d\phi\\ &= \frac{\sin(\pi\tau\Delta f)}{\pi\tau\Delta f}.\end{aligned}$$

* We could have gotten the answer more quickly by just calculating how long it takes a point at the outer edge of the (u, v) plane to move by one dish diameter, namely $(1/\omega)(d/D)$.

A minimum acceptable value of $C = C_{\min}$ implies a maximum phase error $\pi\tau\Delta f = \phi_{\max}$; for example, $C_{\min} = 0.98$ gives $\phi_{\max} = 0.346$ radian. Taking $t = 4N\tau$ for a linear scan at 2 integrations per beam then gives

$$\begin{aligned}
\pi\tau\Delta f &< \phi_{\max} \\
\pi\tau(1.2N\omega D/d) &< \phi_{\max} \\
(1.2\pi/4)t\omega D/d &< \phi_{\max} \\
t &< (4/1.2\pi)(d/D)\phi_{\max}/\omega.
\end{aligned}$$

ALMA Parameters, Worst Case

Taking $C_{\min} = 0.98$ and using the ALMA values of $D = 10\,\text{km}$ and $d = 12\,\text{m}$ then gives $t < 6.06\,\text{sec}$. At the minimum dump time of 16 msec, this allows a scan length of $2N = 379$ beamwidths. However, at long wavelengths the scan length may be limited by the antenna speed; the specification requires accurate pointing (within 1 arcsec) only up to .05 deg/sec, which is 18 arcmin in 6.06 sec. Thus, limiting the worst-case loss to 2% limits the length of a linear scan to 379 beamwidths or 18 arcmin, whichever is less.

There are several ways to extend this limit. First, a coherence loss greater than 2% at the scan edges might be accepted. The loss is a reduction in sensitivity but not in accuracy, since the amount is calculable and correctable. Second, at relatively long wavelengths, pointing need not be as precise as the antenna specification requires, and the specification might be exceeded, so it might be possible to scan faster than .05 deg/sec. Third, more sophisticated scan patterns might be employed in two dimensions, so as to keep the beam within N beamwidths of the phase tracking position for many more than $4N$ integrations.

Very large sources can be observed with multiple scans, each limited to about 6 sec, with negligible loss of efficiency because the time required to initiate a new scan, including changing the phase tracking center, is much less than 6 sec. (If it is arranged that the antenna position at the start of a new scan is very near that at the end of the old scan, then the initiation should take at most 48 msec.)

Conclusion

The strategy of fringe tracking at a constant sky position throughout an OTF scan allows sufficiently large scans with negligible loss of coherence. Whereas this strategy allows by far the simplest implementation, it is the one that should be adopted.

References

[1] L. D'Addario, "Fringe tracking, sideband separation, and phase switching in the ALMA telescope." ALMA Memo No. 287, 2000-Feb-15.

[2] T. Cornwell, "Radio interferometric imaging of very large objects," *Astron. & Astrophys.*, vol 143, p 77, 1988.

[3] A. Thompson, J. Moran, and G. Swenson, *Interferometry and Synthesis in Radio Astronomy.* New York: Wiley, 1986.