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Analysis of Reflective Gratings as Infrared Filters

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Abstract

Reflective diffraction gratings are investigated for use as infrared filters in submillimeter-wave cryogenic receivers, to reduce radiative loading via the dewar window on the 4K stage. The grating diffracts incident IR away from the signal path, back toward the window. Analysis using scalar approximations indicate a single grating filter blocks close to 98% of the incident IR at 300K along the RF signal path, assuming a $\lambda/20$ (15μm) RMS surface deviation and 45 degree incidence. With a blazed grating profile, the filter rejection was found to depend only on the RMS surface deviation and incident angle.

1 Introduction

This memo will study the possible use of a reflective grating as an infrared filter in cryogenically-cooled submillimeter wave receivers, as a means of reducing thermal loading via the dewar window on the 4K stage. The advantages of such a filter are low passband loss, broad bandwidth, and dual use of the grating as a selection mirror for various receivers within the dewar.

Reflective diffraction gratings are widely used in optical and infrared spectroscopy. They have gradually replaced prisms as the primary dispersive element, due to their superior resolution and good efficiency. The properties of diffraction gratings are well understood, and numerous references cover their theory, manufacture, and application in spectroscopic instruments [1, 2, 3, 4, 5].

Diffraction gratings have application in areas other than spectroscopy, such as in high-precision linear metrology [1]. An application that has received little attention are their use as lowpass filters in the far infrared region. A paper by Kudo et. al. [6] proposes the use of reflective gratings to eliminate both short-wavelength infrared radiation and overlapping high diffracted orders in a far infrared spectrometer. The idea of using such a filter to reduce radiative loading on cooled receiver front ends inside a dewar was suggested by James Lamb [7], and is the focus of this memo.

2 Description

A simplified diagram of the optical path inside the receiver dewar is given in Figure 1. The selection mirror/grating normal is tilted 45 degrees from the incident ray, and oriented with the groove edges perpendicular to the incident plane. A different receiver may be selected by rotating the mirror on an axis coincident with that of the incident beam. The grating profile can be either periodic rectangular grooves or triangular ("blazed") facets, as shown in the figure. The groove or facet spacing and height are chosen to keep the RMS surface error to within a small fraction of the shortest wavelength used in the receivers, but large enough to effectively scatter most of the infrared radiation entering the window away from the 4K surfaces of the receiver, preferably back through the window. A blazed grating is particularly useful in this respect, since it has a maximum diffraction efficiency back in the direction of the window when the incident ray angle α and grating blaze angle δ are equal.

3 Analysis

Several assumptions are made to simplify the analysis. First, the polarization of the incident radiation is ignored, and scalar (geometric optics) approximations are used. The scalar theory begins to break down when the groove or facet width approaches the incident wavelength; grating efficiency curves for the two polarizations are significantly different for facet spacing less than three times the wavelength [1]. As will be seen, this is not a serious problem, and the scalar theory can still be used. Second, conductor loss is ignored; this is reasonable for the present purpose. Last, all incident radiation is assumed to be a collimated beam. Whether or not this is strictly true in the infrared depends mainly on the mechanical design of the dewar and telescope optics, which are beyond the scope of this memo.

When a reflection grating is used strictly for filtering, an important property for design is its *relative efficiency*, defined as the ratio of the total power diffracted into an angle β (Figure 1) to the total power incident on the grating, or

$$R(\beta) = \frac{I_{TOT}(\beta)}{\int_\nu M_\nu} = \frac{I_{TOT}(\beta)}{\sigma_B T^4} \tag{1}$$

where $\sigma_B = 5.6697 \times 10^{-8}$ W-m^{-2}K^{-4}, and T is the ambient temperature in Kelvin. I_{TOT} is the absolute intensity function, defined as

$$I_{TOT}(\beta) = \int_\nu M_\nu \cdot IF(\nu, \beta) \cdot BF(\nu, \beta) d\nu \tag{2}$$

where M_ν is the blackbody incident power density (W-m^{-2}Hz^{-1}) versus frequency ν at a temperature T (Kelvin), defined in [8] as

$$M_\nu = \frac{2\pi h \nu^3}{c^2} \left(\frac{1}{e^{h\nu/kT} - 1} \right) \tag{3}$$

where h, c and k are constants. The blackbody curve M_ν for T=300K is plotted in Figure 2.

The functions IF and BF are called the *interference factor* and *blaze function*, respectively, and depend on the physical properties of the grating. The interference factor gives the normalized intensity distribution for a row of N grooves with even spacing σ, versus frequency and angle. It can be derived using geometric optics, and is given from [3] as

$$IF(\nu, \beta) = \frac{\sin^2 N\gamma'}{N^2 \sin^2 \gamma'} \tag{4}$$

where

$$\gamma' = \frac{\pi \sigma \nu}{c} (\sin \beta + \sin \alpha) \tag{5}$$

Note that there is no dependence on either groove spacing or shape; the interference function is the same for all profiles. The blaze function gives the normalized intensity of the diffraction pattern from a single illuminated facet, and is determined from its shape. For the present analysis, a classic "blazed" (triangular) grating profile will be assumed, as in Figure 1. The facet corners are all right angles, and the facet normals tilt back toward the dewar window. For this profile, the blaze function is given in [3] by

$$BF(\nu, \beta) = \frac{\sin^2 \gamma}{\gamma^2} \tag{6}$$

where

$$\gamma = \frac{\pi\sigma\nu\cos\delta}{c}[\sin(\beta-\delta)+\sin(\alpha-\delta)] \tag{7}$$

With the above relations, the integral in Eq. (2) can be evaluated numerically over a range of angles β, given the blaze angle δ, facet spacing σ, incident beam angle α, number of facets N, and a range of ν wide enough to include all of the 300K blackbody curve in Figure 2. While this can be done in principle, there are a number of practical difficulties. One is the mathematical nature of Eq. (4); as N gets large (500–1500 would be typical), the number of local maxima from all the pairs of interfering slits over all orders of diffraction becomes huge. In addition, these peaks are extremely narrow and sharp, and much finer resolution in β (i.e., more points) is required to do the integration with any reasonable accuracy. Thus the computation gets drastically slower as N increases, which makes it difficult to evaluate the effect of the other variables (δ and σ) on I_{TOT}.

A more productive approach is to look at the single case when $\beta = -\alpha$ (as in specular reflection from a plane mirror). This is of primary interest, as it points directly into the receiver feed. From Eq. (5), one can see that $\gamma' = 0$. It then readily follows that in the limit $\gamma' \to 0$, $IF(\nu,\beta) \to 1$. Thus, the value of I_{TOT} becomes independent of N (and far easier to calculate). The blaze function also is somewhat simplified; for $\beta = -\alpha$, Eq. (7) reduces to

$$\gamma = \frac{-\pi\sigma\nu\cos\alpha}{c}\sin(2\delta) \tag{8}$$

Figure 3 shows the computed values of I_{TOT} over a wide range of facet spacing (pitch), for 45 degree incidence, a blaze angle of 45 degrees, and blackbody temperature T=300K. For extremely small facets, the grating approximates a plane mirror, and virtually all of the incident power is reflected into the feed, as expected ($\sigma_B T^4$ = 459.25 W-m^{-2}). At the other extreme, a substantial amount of power incident on a coarse grating diffracts into higher orders, at angles away from the receiver feed. Clearly, longer facets will yield more rejection of the incident infrared radiation; the limit in this direction is on how much deviation in the surface can be tolerated at the highest receiver frequency. Obviously, the blaze angle can be reduced to keep the surface deviation down for longer σ, but this also shifts the peak of the blaze function BF toward $\beta = -\alpha$. If the surface deviation is held constant, are there optimum values for δ and σ that would minimize I_{TOT}?

To pursue this further, the RMS surface deviation of a blazed grating in terms of δ and σ is calculated. The RMS deviation D is expressed by

$$D = \left[\frac{1}{\sigma}\int_0^{\sigma}[f(x)]^2 dx\right]^{1/2} \tag{9}$$

where $f(x)$ is the facet profile, a piecewise linear function defined as

$$f(x) = \begin{cases} x \cdot \tan\delta & \text{for } 0 \le x \le x' \\ (\sigma - x)\cdot\cot\delta & \text{for } x' \le x \le \sigma \end{cases} \tag{10}$$

Evaluating Eq. (9) yields the following expression for D in terms of δ and σ:

$$D = \frac{\sigma\sin(2\delta)}{2\sqrt{3}} \tag{11}$$

Solving Eq. (11) for σ and substituting into Eq. (8) gives

$$\gamma = \frac{-2D\sqrt{3}\cdot\pi\nu\cos\alpha}{c} \tag{12}$$

Note that in the above expression for γ, δ and σ have dropped out completely! Thus, for a blazed grating filter, *the ultimate rejection is limited by the allowable RMS surface deviation; there is no optimum pair of δ and σ.* To quantify this with a practical example, Figure 4 gives a plot of I_{TOT} versus D, assuming 45 degrees incidence at 300K. Assuming the grating surface deviation is 15μm ($\lambda/20$ at 1 THz), the filter rejection will be approximately $(1 - 10/459.25)$, or **97.8%**.

Since D is typically a fixed maximum, corresponding to a fraction of the shortest received wavelength, the only parameter left that can affect I_{TOT} is the incident beam angle α. Figure 5 shows the change in I_{TOT} versus incident angle, for $D = 15\mu$m. The choice of α in a practical receiver will naturally be limited by mechanical design constraints.

If we fix D at 15μm as in the above example, it would be of interest to calculate the facet spacing for practical blaze angles. Solving Eq. (11) for σ, its minimum value ($\delta = 45°$) is approximately $3.46D$, or 52μm. At T=300K, the blackbody curve peaks near $\lambda = 10\mu$m; thus the facet spacing is more than five times the wavelength at this peak. Hence it is reasonable to assume that over much of the blackbody curve, the polarization effects alluded to at the beginning of this section are minimal.

Lastly, a computation of I_{TOT} over a broad range of diffraction angles was performed by direct integration of Eq. (2), with $\alpha = \delta = 45°$, $\sigma = 52\mu$m, and N=1000. The plot is shown in Figure 6. The resolution is not adequate to show the very fine structure of sharp peaks and nulls from diffraction, but does show the broad peak near the angle of incidence. There is very little incident power diffracted into the regions to either side of $\beta = -45°$, which is good.

4 Conclusions

The above analysis demonstrates that reflective gratings can be used as infrared filters with good performance. At 45 degree incidence, almost 98is obtained. Advantages of a grating over a conventional IR filter are that it can be integrated with the selection mirror, saving space in the dewar, and is free of dielectric and mismatch losses. The main disadvantage is the difficulty and high cost of ruling the grating, particularly if on a non-planar surface (e.g., an ellipsoid). Grating replicas could be used for production mirrors, at a substantial cost savings. However, the ability of the replica grating (essentially a glass-backed, aluminized resin block) to survive repeated temperature cycling inside a dewar is unknown at the present time.

References

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[8] W. Wolfe and G.J. Zissis (ed.), *The Infrared Handbook*, Environmental Research Institute of Michigan, 1978

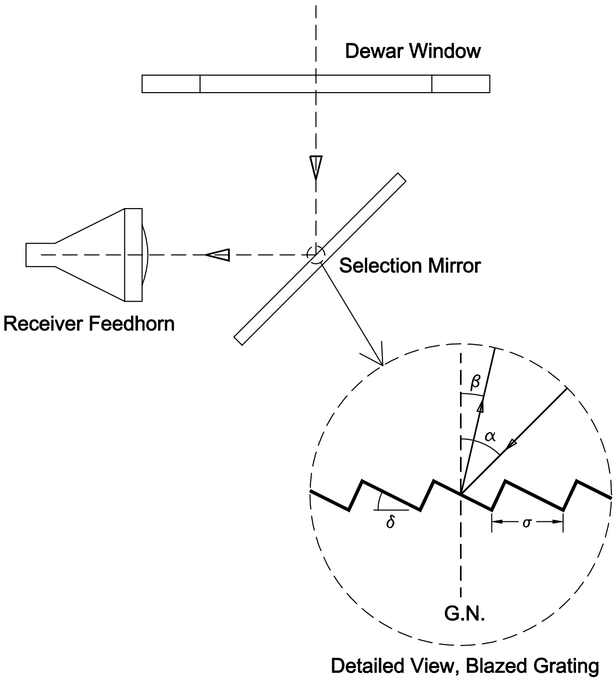

Figure 1: Receiver Selection Mirror/Diffraction Grating Filter

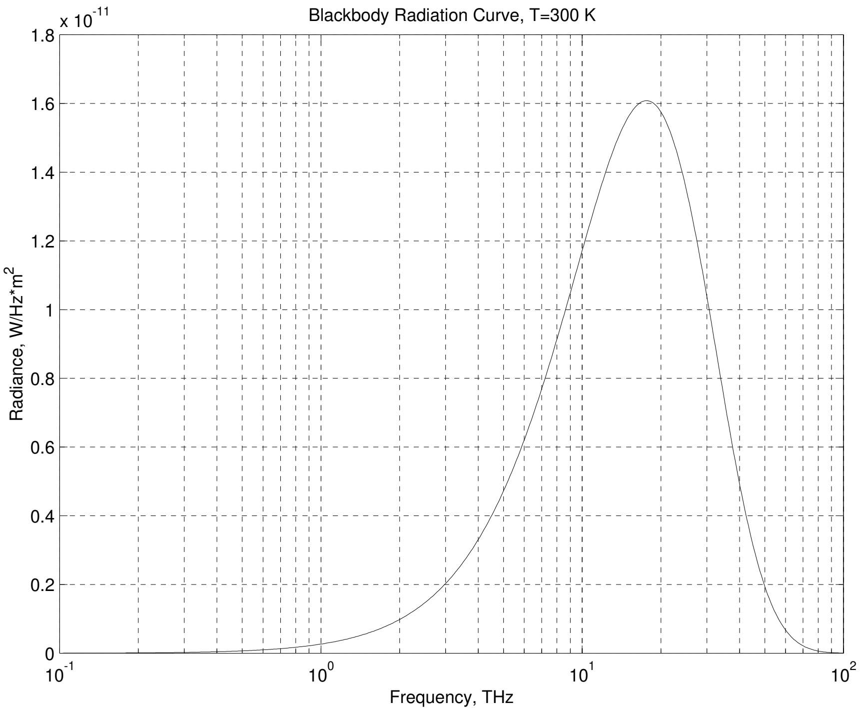

Figure 2: Room Temperature Blackbody Radiation Curve vs. Frequency

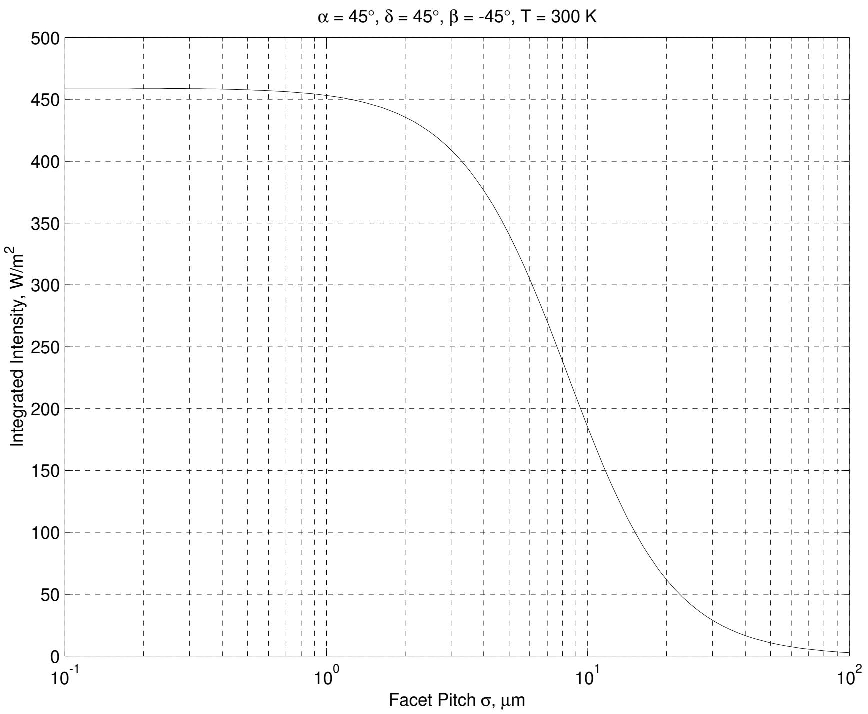

Figure 3: Grating Filter Output Intensity (into feed) vs. Facet Spacing

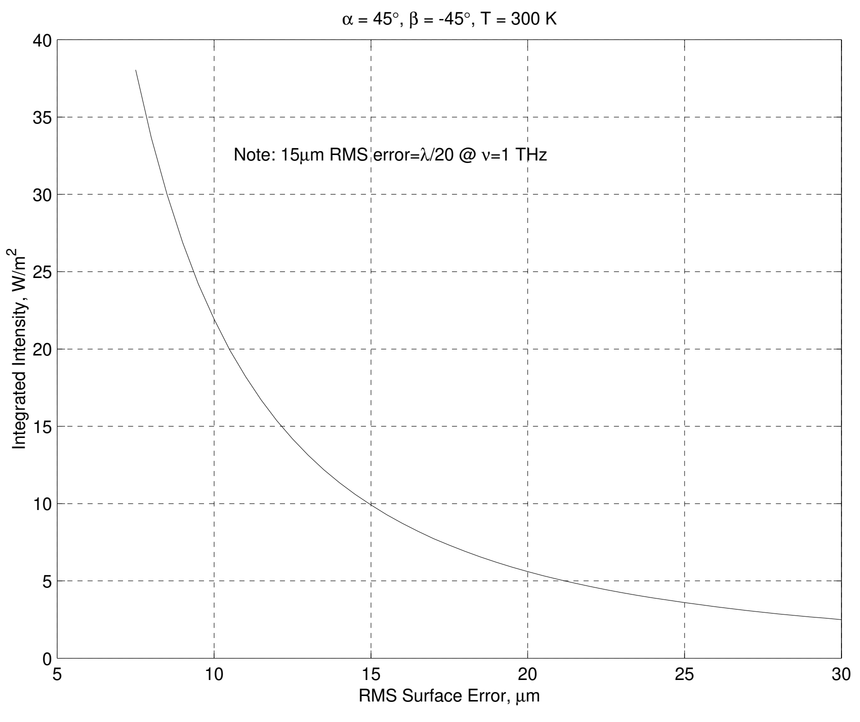

Figure 4: Output Intensity (into feed) vs. RMS Deviation, 45° Incidence

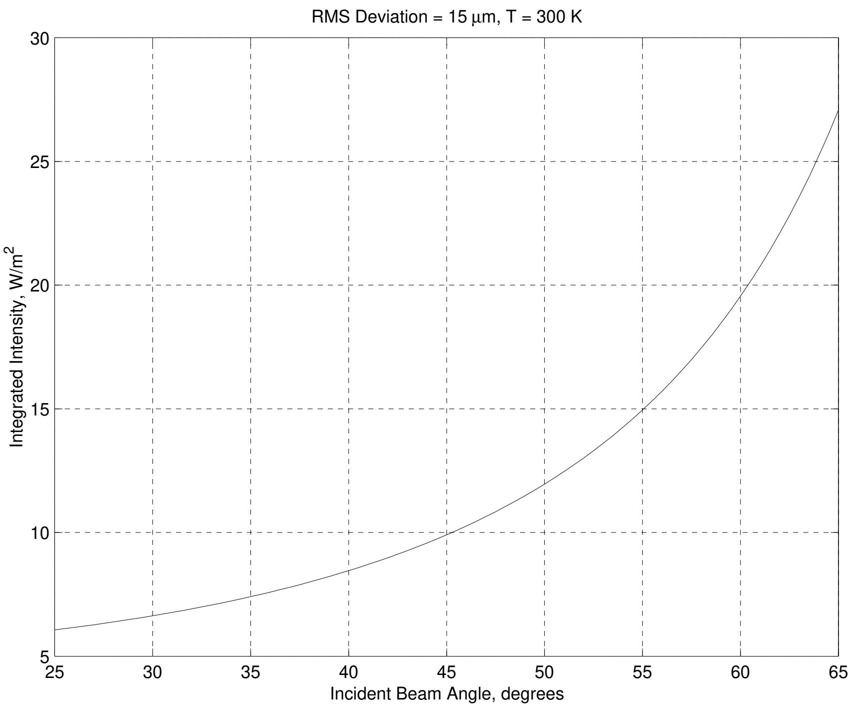

Figure 5: Output Intensity (into feed) vs. α, Fixed RMS Deviation

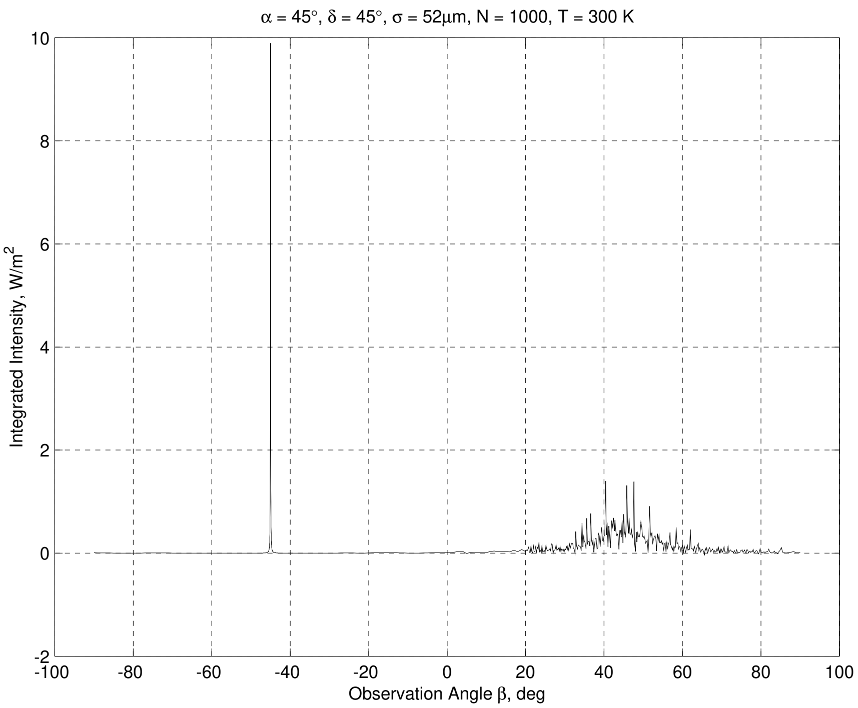

Figure 6: Output Intensity vs. β, 45° Incidence, 15μm RMS Deviation