# MMA Memo 239: Hybrid composite arrays: I. The inner and outer hybrids

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**Abstract**

A design for the MMA is considered that is a composite of three configurations of different shape and size, termed the compact, intermediate and extended configurations, and the possibility is investigated of hybridizing these configurations in order to provide the ability to zoom. The quality of the beams resulting from various hybridization strategies applied to the inner hybrids (those between the compact and intermediate configurations) is found to depend strongly on the details of the geometrical design of the configurations and on where exactly the antennas are sited, but the best of the hybrids have excellent beam profiles with simple, bell-shaped main lobes and very weak sidelobes. There is less variation in the quality of the outer hybrids, which have clean main lobes and show a progressive increase in sidelobe level as antennas are moved from the intermediate to the extended configuration; the interesting result is found that the array may be built with a wide gap between these two configurations without adversely affecting the quality of the beams, enabling not only a significant reduction in infrastructure costs but also the accommodation of the array to the topography of the site at Chajnantor. The study is based on a continuum approximation and must be checked against a calculation based on a more detailed discrete model, but the results support the view that it may be possible to design the MMA as a telescope with a full zoom capability having beam profiles of acceptable quality over the entire zoom range.

## 1 Introduction

In a previous paper (Webster 1998, Paper I), a new and general approach was presented to the design of the intermediate configurations of the MMA. It is based on two ideas, the first of which is to take the stations for the antennas to be distributed widely, with the density dropping off as the inverse square of the distance from the centre of the array, while the second is to adopt a continuum approximation for the distribution of antennas and stations in order to simplify the investigation of the properties of the synthesized beam. At any given time the antennas are taken to be sited within a circular annulus, and since there are many such annuli possible on the stations of an inverse-square distribution there are many choices of configuration available that might be useful for making observations (c.f. Viallefond 1998). A particularly important feature of these configurations is the size distribution, which is sufficiently dense that an annulus can always be found with a beamwidth within about one percent of any desired value in the appropriate range, giving the telescope a 'zoom' capability (c.f. Conway 1998). It was found that for a wide range of the key parameter $R \equiv a/b$ (where $a$ is the radius of the outer edge of the annulus containing the antennas and $b$ is the radius of the inner edge) the synthesized beam has excellent properties, including a low sidelobe level.

The intermediate configuration is just one of three that make up the MMA in the design under consideration, the other two being termed the compact and extended configurations. The three configurations differ in size, as the names imply, but they also differ in shape as a result of a strong dependence of the performance criteria for the telescope on the size of the configuration, the compact configuration being optimized for sensitivity to extended structure of low surface brightness, the intermediate for the quality of the beam profiles and for the ability to zoom, and the extended for high angular resolving power. These criteria result in completely different designs. The stations of the compact configuration are best sited as close together as possible within an approximately circular region; those of the intermediate configuration are better distributed widely, perhaps with an inverse-square density distribution; and those of the extended configuration are best sited on a ring in the shape of a circle or Reuleaux triangle. The disparity in shape between the configurations appears to be a new departure in the design of large aperture-synthesis telescopes, where it is more usual to find configurations of one shape provided in several different sizes, and the term *composite array* is adopted here for a telescope made up of dissimilar configurations.

The composite nature of the new type of telescope influences how it would be used, because an astronomer wishing to make observations at any given wavelength would have an incomplete range of beamwidths to choose from, made up of the narrowest beam, obtained by siting all the antennas on the most extended configuration, the broadest, by siting them on the most compact, and a range of intermediate values by siting them on the intermediate configuration. The latter range is densely covered by virtue of the ability to zoom, but it is of limited extent and does not reach to the narrowest and broadest beams. The ability to zoom is an attractive feature of this kind of telescope but it is only available for the intermediate configuration, leaving two wide gaps in the coverage near the extremities of the range.

It is possible, however, to fill these gaps by operating the telescope on occasion in a way that has not been considered in the design of the three individual configurations, namely as a hybrid, in which some of the antennas are placed on the stations of one configuration and the remainder on the stations of another. Hybridization of this kind always brings with it the ability to zoom because the beamwidth changes progressively as antennas are transferred from one configuration to another of different size, whatever the detailed design of those configurations. For a telescope with as many antennas as the MMA, moreover, the size of the configuration changes by only a small factor when just one of them is transferred, allowing the beamwidth to be controlled very finely.

While such a *hybrid composite array* would automatically have a zoom capability over the widest possible range of beamwidths, there is unfortunately no guarantee that the profiles of the synthesized beams of the hybrids would be good enough for the purpose of making astronomical observations, the basic configurations having been optimized in isolation without the possibility of hybridization being taken into account. A telescope that does have the capability to zoom continuously from the narrowest to the broadest beams while maintaining beam profiles of high quality over the entire range would be endowed with unprecedented flexibility and power, and we are concerned here with establishing whether such a telescope can be designed.

The paper divides naturally into two sections, one for the *inner hybrids* between the compact and intermediate configurations and the other for the *outer hybrids* between the intermediate and extended configurations. These two types of hybrid require separate treatment, not only because they involve different pairs of configurations but also because they have to meet different criteria. The inner hybrids are of limited spatial extent and the resulting high sensitivity permits observations to be made in snapshot mode, for which purpose it is vital that the beams have low instantaneous sidelobe levels. The outer hybrids, by contrast, are of considerable spatial extent and the resulting low sensitivity necessitates long integrations in earth-rotation synthesis mode, for which low instantaneous sidelobe levels are less important. The demands are therefore higher on the inner hybrids, and the quality of their instantaneous beam profiles is crucial.

## 2 The calculation

In order to keep the investigation simple the continuum model of Paper 1 is adopted without change. It is therefore assumed that there exists a continuous function of position $\rho_{\mathrm{S}}(r)$ giving the density of stations for the whole array, which is circularly symmetrical and is a function only of $r$, the radial distance from the centre of the array. It is also assumed that there exists a second continuous function $\rho_{\mathrm{A}}(r)$ (where $0 \leq \rho_{\mathrm{A}}(r) \leq \rho_{\mathrm{S}}(r)$) giving the surface density of antennas. The three configurations of the composite array are modelled differently: the compact configuration, in which the stations are sited as close together as possible, is modelled as a uniform disc of radius $r_{\mathrm{C}}$; the intermediate configuration is modelled, as in Paper 1, as an inverse-square distribution between inner and outer radii of $r_{\min}$ and $r_{\max}$ respectively; and the extended configuration is taken to be a ring, modelled as a circle of radius $r_{\mathrm{E}}$. $\rho_{\mathrm{S}}$ is therefore

$$\rho_{\mathrm{S}}(r) = \begin{cases} \rho_{\mathrm{C}} & 0 \leq r \leq r_{\mathrm{C}} \\ \rho_0 r^{-2} & r_{\min} \leq r \leq r_{\max} \\ \rho_{\mathrm{E}} \delta(r - r_{\mathrm{E}}) & r \geq r_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $r_{\mathrm{C}} \leq r_{\min}$ and $r_{\max} \leq r_{\mathrm{E}}$. The density constants $\rho_{\mathrm{C}}$ and $\rho_{\mathrm{E}}$ are determined by the requirement that the compact and extended configurations consist of $N_{\mathrm{A}}$ stations each

$$\rho_{\mathrm{C}} = \frac{N_{\mathrm{A}}}{\pi r_{\mathrm{C}}^2} \tag{2}$$

$$\rho_{\mathrm{E}} = \frac{N_{\mathrm{A}}}{2\pi r_{\mathrm{E}}} \tag{3}$$

where $N_{\rm A}$ is the number of antennas, while $\rho_0$ is determined by the considerations in Paper 1. The total number of stations is taken to be $N_{\rm S} = 4N_{\rm A}$, so that there are $2N_{\rm A}$ stations in the intermediate configuration.

$\rho_{\rm A}$ is determined by how the antennas are divided over the two adjacent configurations. If the latter are labelled 1 and 2 respectively then

$$\rho_{\rm A} = (1-f)\rho_1(r,f) + f\rho_2(r,f) \tag{4}$$

where $\rho_1$ and $\rho_2$ are the densities of the antennas on the two configurations individually and $f$ is a parameter that determines how the antennas are divided between the two configurations, $0 \leq f \leq 1$. The convention is adopted that $\rho_1$ refers to the more compact of the two configurations being hybridized and $\rho_2$ to the more extended. For intermediate values of $f$, the density of antennas on any configuration need not be proportional to the density of stations, and the parameter $f$ therefore appears explicitly in $\rho_1$ and $\rho_2$.

## 2.1 The inner hybrids

The inner hybrids involve the compact and intermediate configurations, and the different ways considered here of siting the antennas on them are shown in Fig. 1. The first three panels show the results of a) removing the antennas uniformly from the compact configuration as $f$ increases, b) removing them from the centre outwards and c) removing them from the circumference inwards. The last two panels how the results of d) adding the antennas to the intermediate configuration from the inner edge progressively outwards and e) adding them uniformly across the innermost solid annulus without preference as to radius.

There are other degrees of freedom that must be specified before the beam profiles may be calculated. The first is the value of $R$, the invariant parameter describing the thickness of solid annuli on the intermediate configuration, for which values of about $R = 4$ may be appropriate for the MMA, as discussed in Paper 1. The second is the size of any gap between the compact configuration and the intermediate, which is specified by the ratio $r_{\rm min}/r_{\rm C}$.

There are many different permutations of these degrees of freedom, and it is not possible to calculate the results for all of them and present them here. The procedure adopted instead is to start by adopting one particular model that is regarded as standard and to proceed by deriving new models from it by changing just one of the degrees of freedom. The standard model

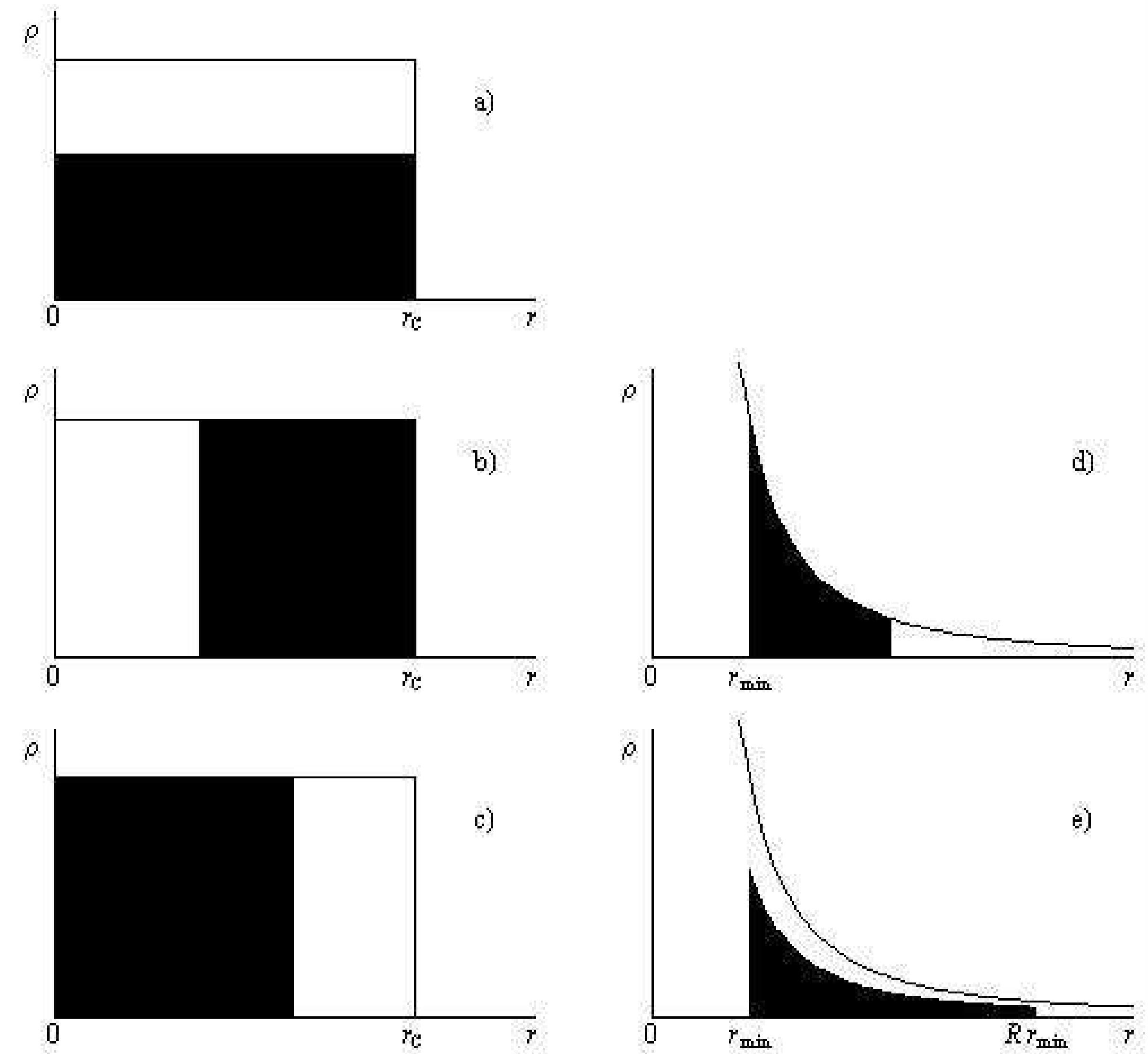


Figure 1: Different ways of distributing the antennas over the stations of the inner configuration (left) and the intermediate configuration (right) when generating inner hybrids. The thin line in each case gives the density of stations $\rho_{\rm S}$ as a function of $r$, and the shaded curve gives the density of antennas $\rho_{\rm A}$ when the configuration is only partially populated. In a) and e) the antennas are distributed uniformly over the stations, with no preference as a function of $r$; in b) they have been removed only from a central hole, and in c) only from the circumference, leaving the remaining stations fully populated in both cases; in d) the sites of the intermediate configuration have been filled progressively from the inner edge.

is obtained by combining cases a) and d) in Fig. 1, in which the compact configuration is depleted uniformly while the intermediate configuration is populated from the inside edge outwards; the value $R = 4$ is adopted and it is supposed that there is no gap between the outer edge of the compact configuration and the inner edge of the intermediate one ($r_{\mathrm{min}} = r_{\mathrm{C}}$). The beam profiles are calculated by means of equation 5 of Paper 1 and are given in Fig. 2 for a range of values of the hybridization parameter $f$. The wavelength and the overall size of the array enter the calculation of the beam profile, but in a simple way which affects only the scaling of the variable $\theta$. The results were therefore simplified by setting $r_{\mathrm{C}} = 1$ and $\lambda = 1$, and the results for other values may be derived by taking the unit of $\theta$ to be $\lambda/r_{\mathrm{C}}$ radians.

The first degree of freedom to be varied is the annular ratio $R$. Fig. 3 gives the profiles for $R = 6$ and Fig. 4 those for $R = 3$. Next, the method by which the compact configuration is depleted is changed from the unifom depletion of the standard model to case b) (depletion from the centre outwards) in Fig. 5 and to case c) (depletion from the circumference inwards) in Fig. 6. The corresponding change in the way the stations of the intermediate configuration are filled (uniformly across the smallest solid annulus) is given in Fig. 7, and finally a gap of a factor of two is introduced between the compact and intermediate configurations ($r_{\mathrm{min}} = 2r_{\mathrm{C}}$) in Fig. 8.

## 2.2 The outer hybrids

There are fewer permutatations for the outer hybrids because the choice of outer ring is unique and because only two alternatives are considered for the way the intermediate configuration is depleted. They are broadly similar to cases already depicted in Fig. 1 for the inner hybrids: the first is termed case f) and consists of depletion from the inside out, while the second is termed case g) and consists of depletion uniformly across the final solid annulus from max/$R$ to max. The previous strategy is adopted of choosing a model that is regarded as standard and then deriving new models from it by changing just one of the degrees of freedom. The standard model is case f) with $R = 4$ and no gap (i.e. with $r_{\mathrm{E}} = r_{\mathrm{max}}$) and the results are given in Fig. 9. Fig. 10 gives the results when $R = 6$, and Fig. 11 when $R = 3$. Fig. 12 gives the results for case g) and $R = 4$, and finally Fig. 13 gives the results for the standard case but with a gap of a factor of 2 between the outer edge of the intermediate configuration and the outer ring, $r_{\mathrm{E}} = 2r_{\mathrm{max}}$.

Throughout these calculations for the outer hybrids, the scaling laws

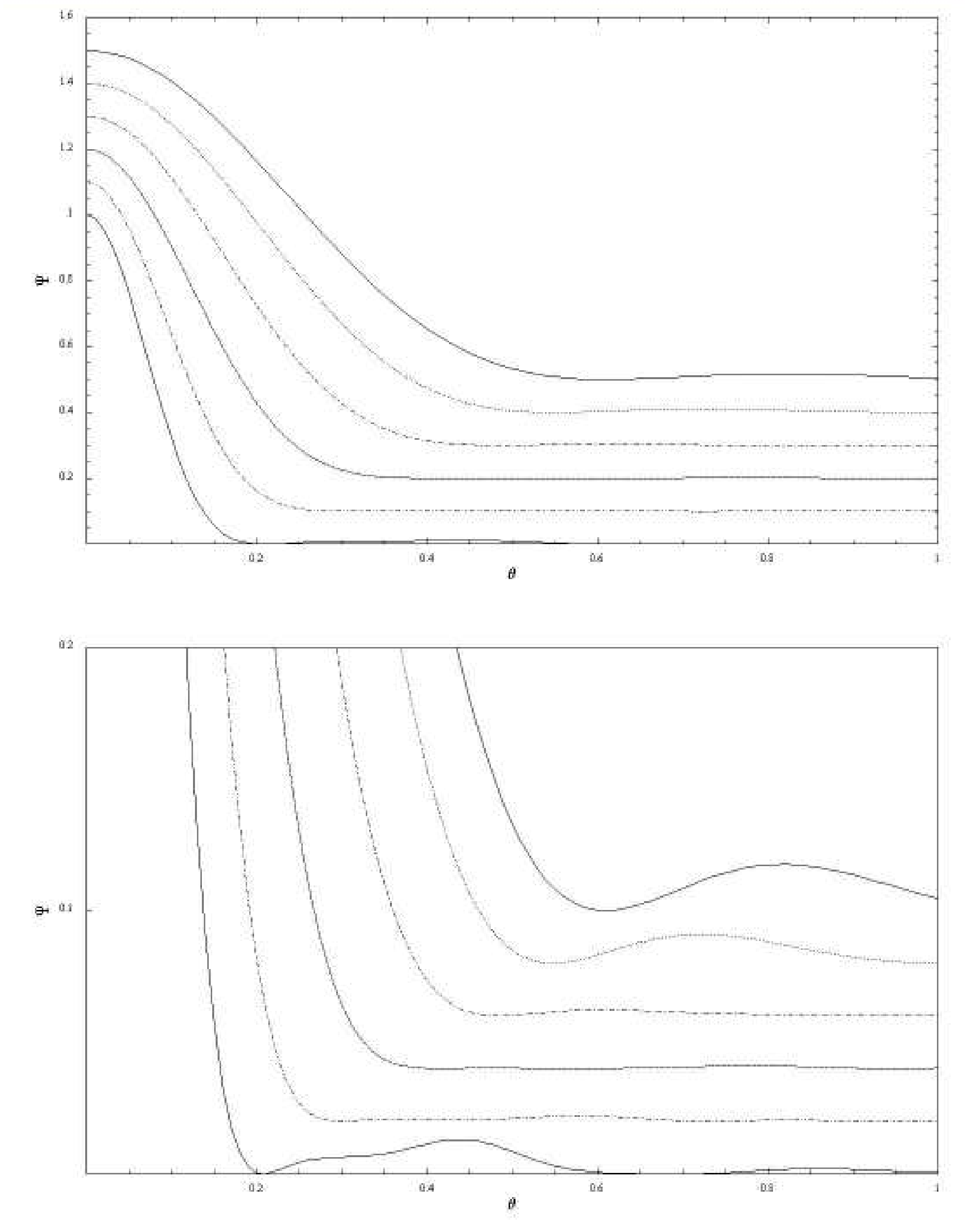


Figure 2: The beam profiles of inner hybrids of type a) + d), with $R = 4$ and no gap (i.e. with $r_{\mathrm{min}} = r_{\mathrm{C}}$). In this and subsequent figures the curves are for $f = 0$, 0.2, 0.4, 0.6, 0.8 and 1. The curves are displaced vertically for clarity, with that for $f = 0$ (i.e. with all the antennas on the compact configuration) at the top. The curves in the lower panel are the same as in the upper, but on an extended vertical scale in order to display the weak sidelobes.

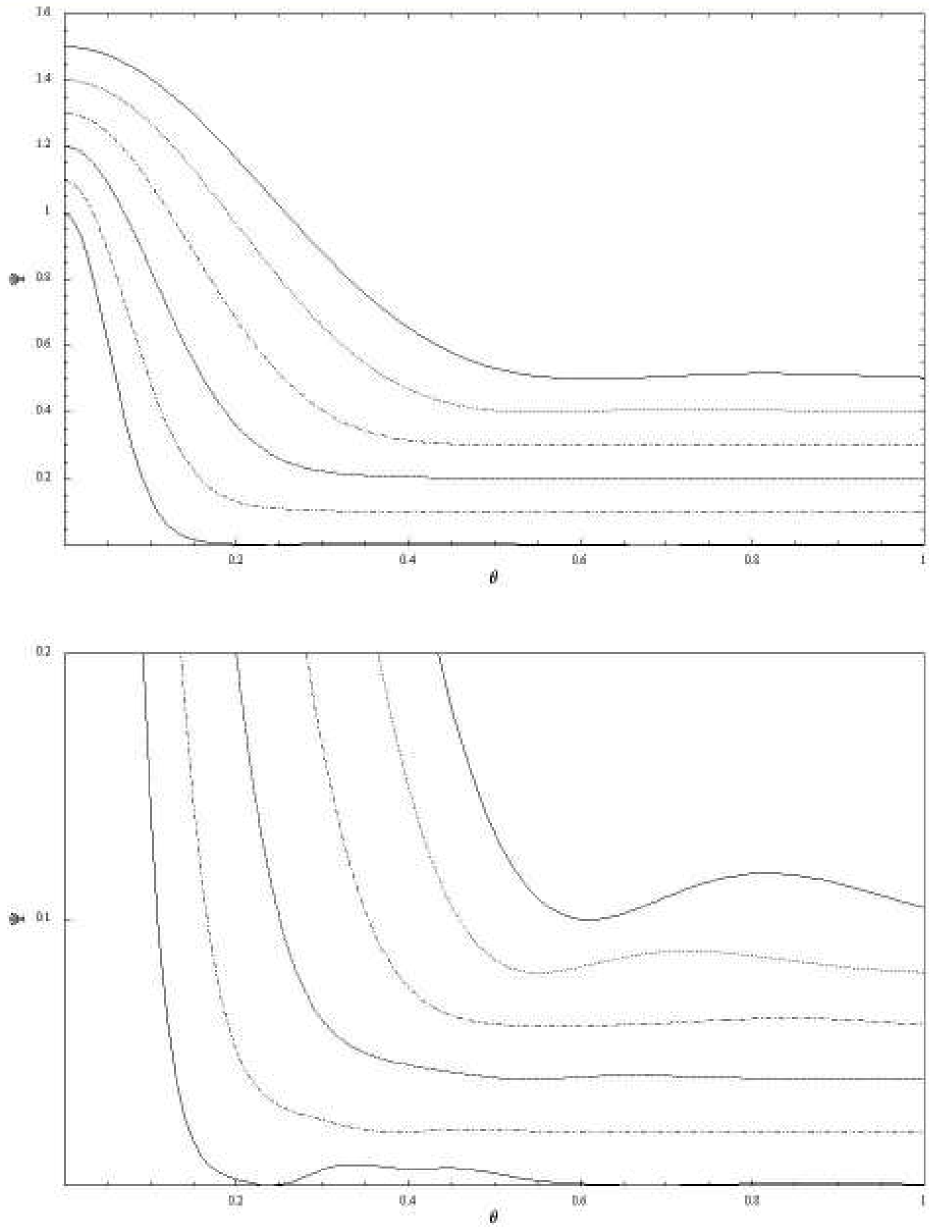

Figure 3: The beam profiles of inner hybrids of type a) + d), with $R = 6$ and no gap (i.e. with $r_{\min} = r_C$).

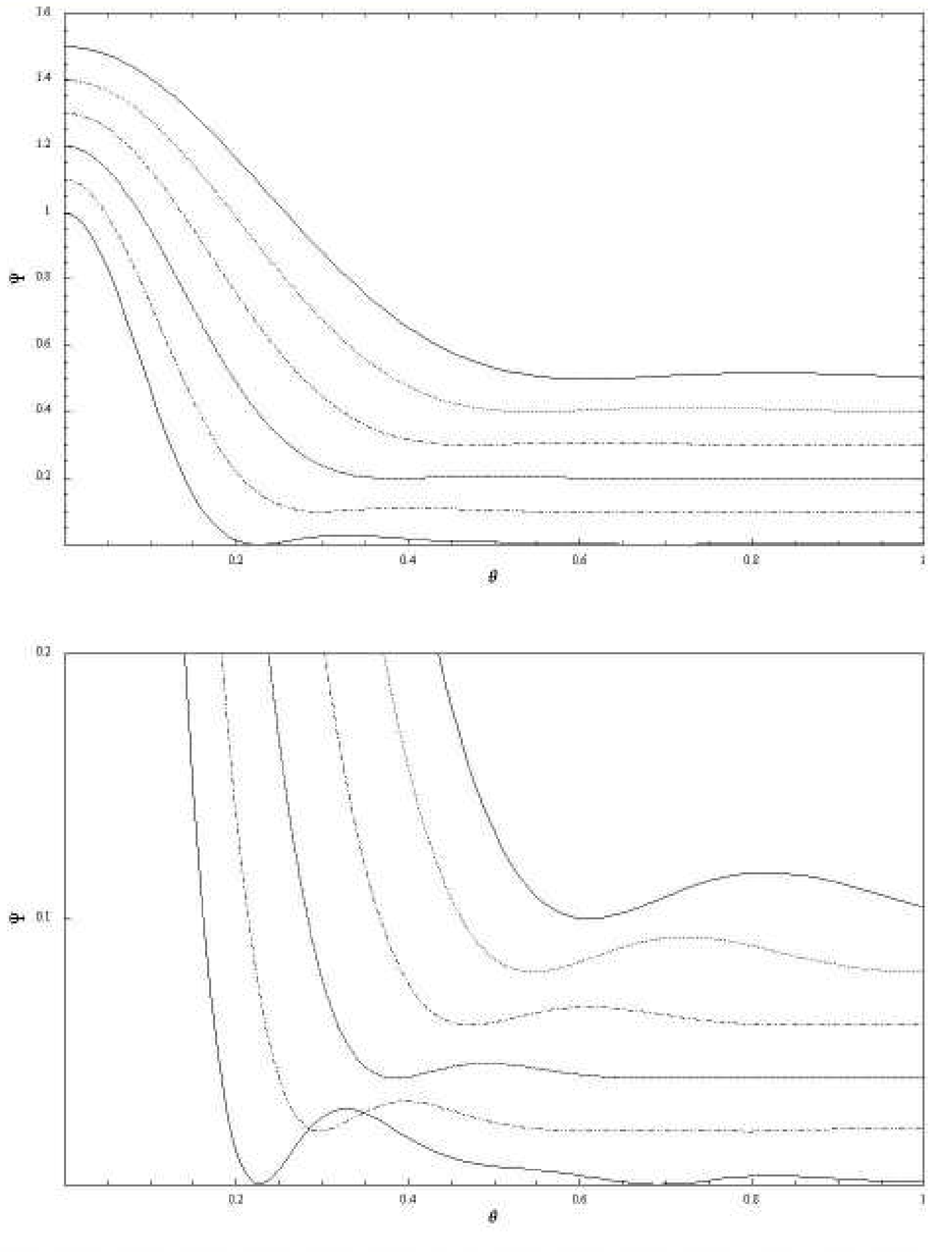


Figure 4: The beam profiles of inner hybrids of type a) + d), with $R = 3$ and no gap (i.e. with $r_{\min} = r_{\rm C}$).

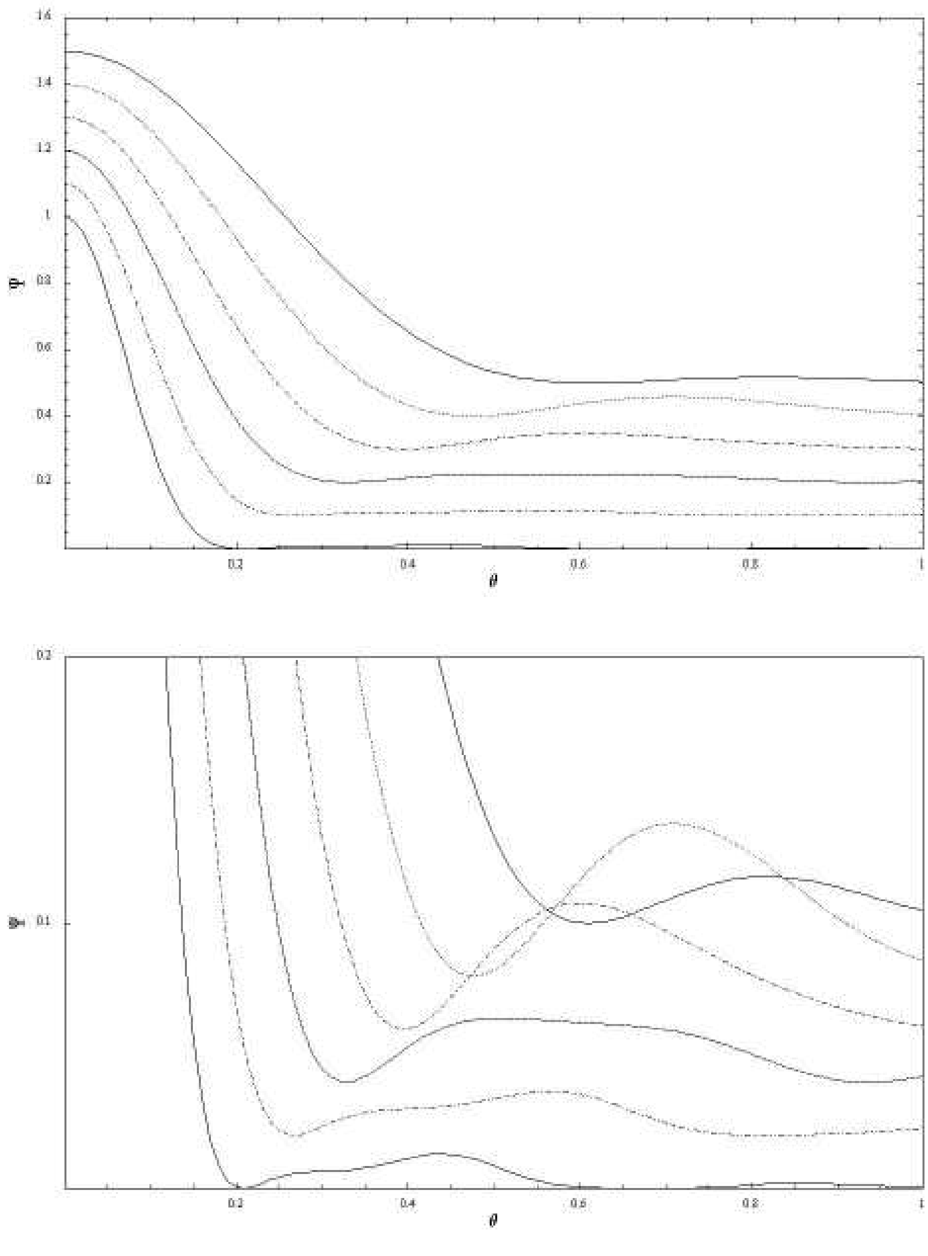


Figure 5: The beam profiles of inner hybrids of type b) + d) with $R = 4$ and no gap (i.e. with $r_{\min} = r_C$).

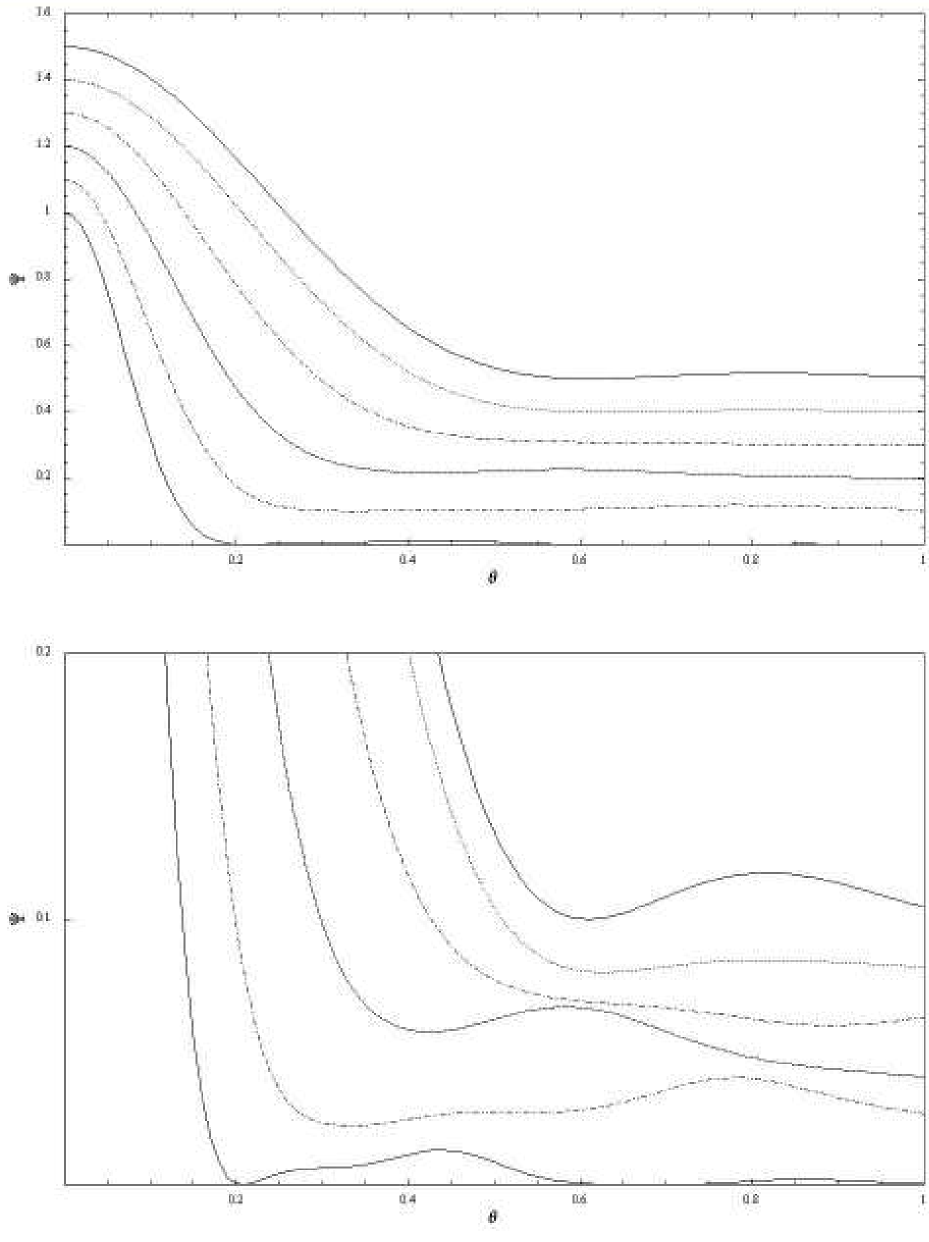


Figure 6: The beam profiles of inner hybrids of type c) + d), with $R = 4$ and no gap (i.e. with $r_{\rm min} = r_{\rm C}$).

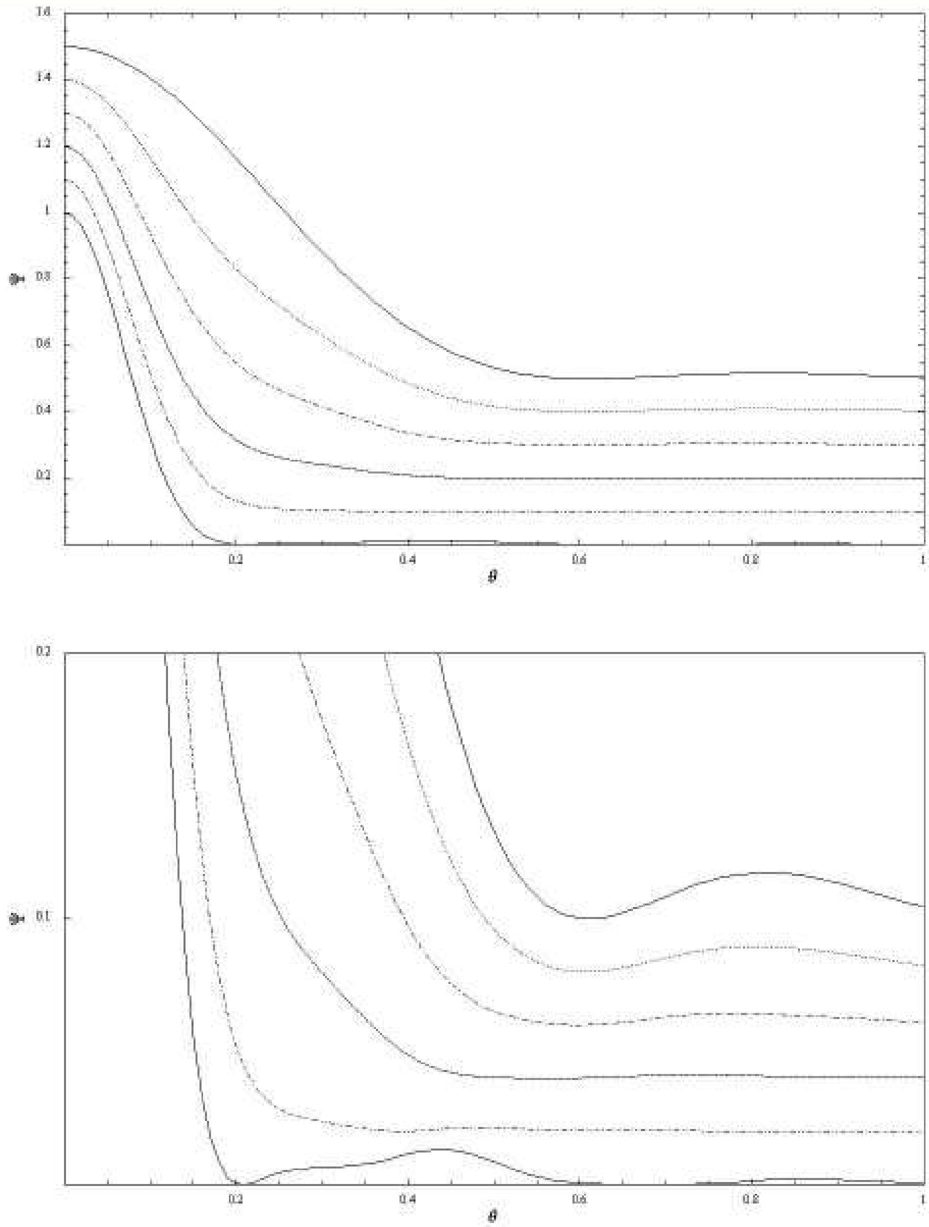


Figure 7: The beam profiles of inner hybrids of type a) + e), with $R = 4$ and no gap (i.e. with $r_{\min} = r_{\mathrm{C}}$).

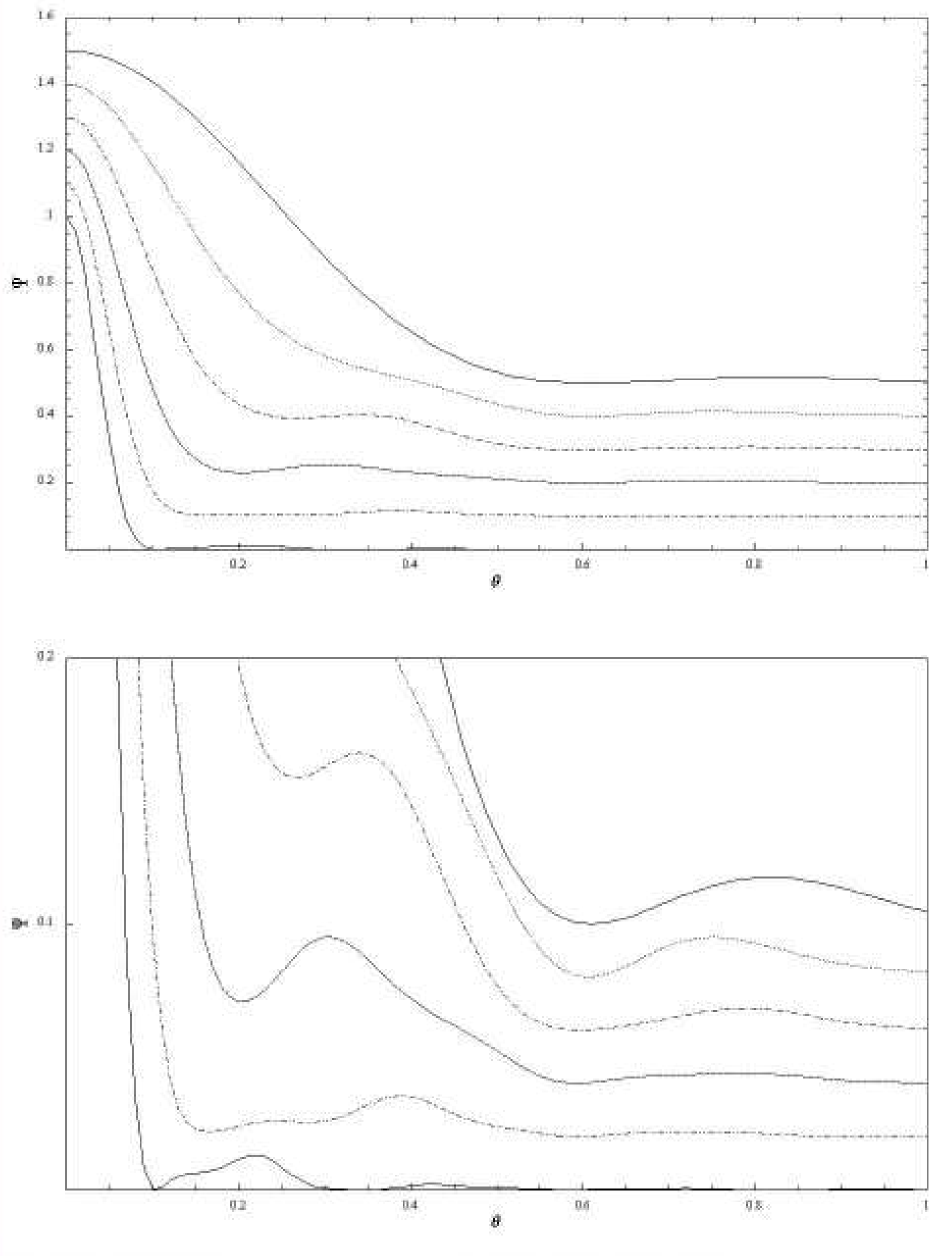

Figure 8: The beam profiles of inner hybrids of type a) + d), with $R = 4$ and a gap of a factor of two (i.e. with $r_{\rm min} = 2r_{\rm C}$).

have been applied by adopting $r_{\mathrm{E}} = 1$ and $\lambda = 1$, so the profiles for other values may be derived by taking the unit of $\theta$ to be $\lambda / r_{\mathrm{E}}$ radians.

# 3 Discussion

## 3.1 The inner hybrids

It was mentioned in the Introduction that hybridization automatically bestows a zoom property on the array, and this property is manifested in all the diagrams related to the inner hybrids (Figs 2 – 8) as a monotonic decrease in the diameter of the beam as $f$ increases; the decrease is the direct result of the progressive increase in the average diameter of the array. The rate at which the beamwidth decreases with $f$ is relatively steady in most of the cases studied, the exceptions being in Figs 7 and 8 where the decrease is most rapid for small values of $f$ and is slower for large.

Another feature common to most of the results is the clean, simple structure of the forward lobe of the synthesized beam. In most cases this is an uncomplicated bell-shaped curve that should be ideal for making astronomical observations. The principal exception is that in Fig. 7, where the hybrids for $f$=0.2, 0.4 and 0.6 show a broad shoulder, the forward lobe appearing to consist of a superposition of two bell-shaped curves, one of them narrow and the other broad.

The greatest divergence between the various results is in the most important property, namely the level of sidelobes. The levels are relatively high in Figs 5 and 8, moderate in Figs 4 and 6, and low in Figs 2, 3 and 7. It is remarkable how low the sidelobes are in the best cases: in Figs 2 and 3 the sidelobes of the hybrids are actually weaker than those of the basic configurations out of which they are hybridized.

It was found in Paper I that the sidelobe levels of solid annuli on inverse-square arrays decrease as $R$ is increased, and the same may be seen to be true of the inner hybrids in Figs 2, 3 and 4.

## 3.2 The outer hybrids

Many of the general properties of the outer hybrids resemble those of the inner. The beamwidths decrease monotonically as $f$ increases, the rate of decrease being fairly uniform in most cases, with the exception of Fig 13. The forward lobes are once again simple bell-shaped curves.

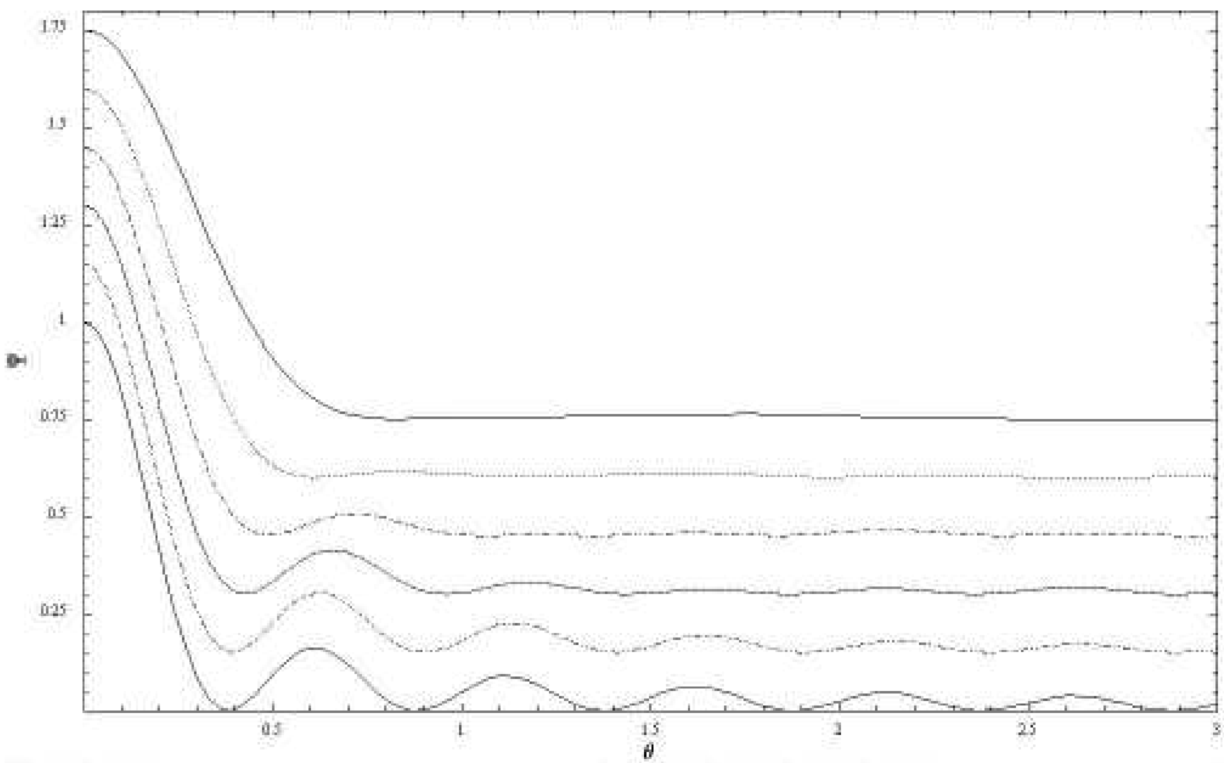


Figure 9: The beam profiles of outer hybrids of type f), with $R = 4$ and no gap (i.e. with $r_{\mathrm{E}} = r_{\max}$).

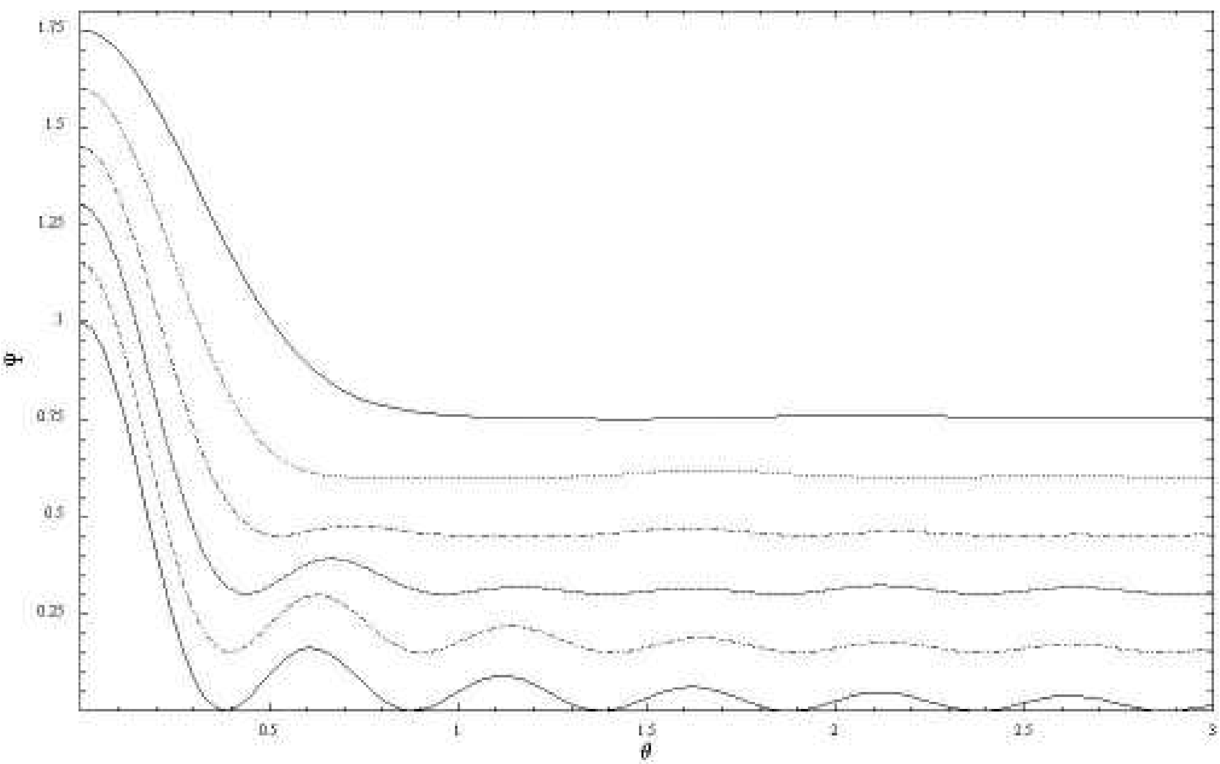


Figure 10: The beam profiles of outer hybrids of type f), with $R = 6$ and no gap (i.e. with $r_{\mathrm{E}} = r_{\max}$).

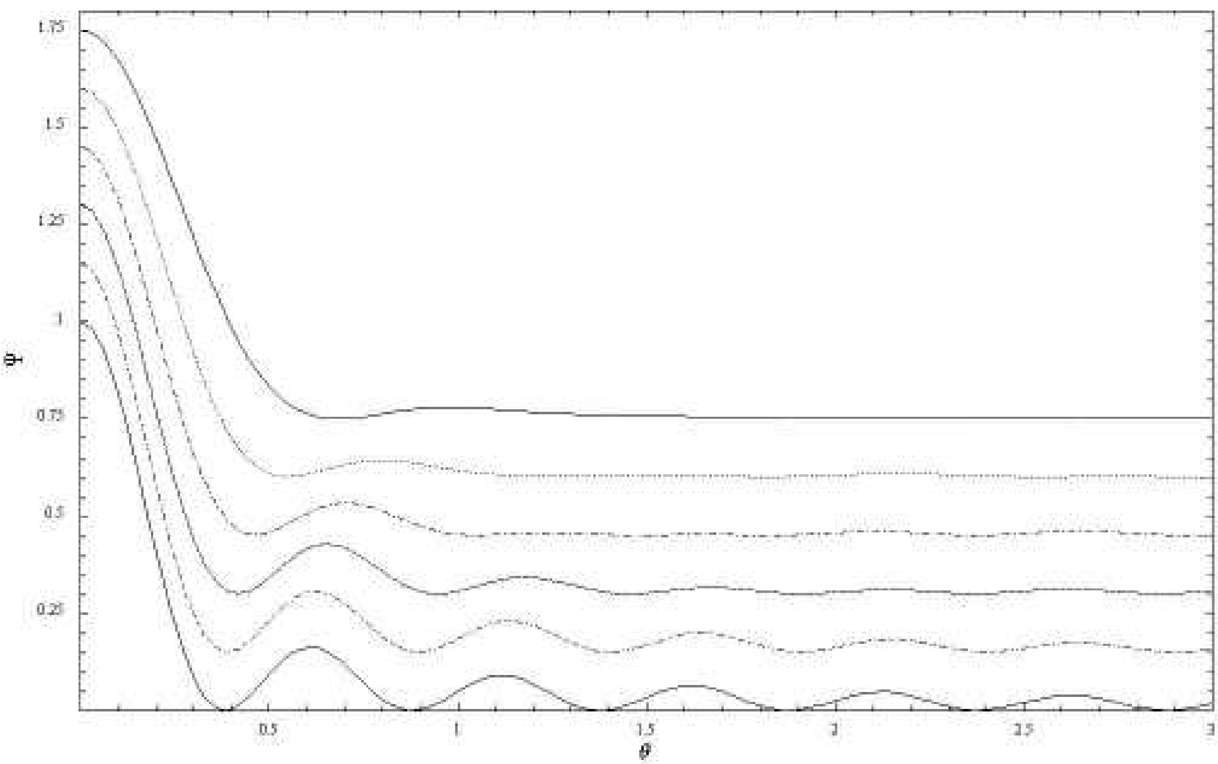


Figure 11: The beam profiles of outer hybrids of type f), with $R = 3$ and no gap (i.e. with $r_{\mathrm{E}} = r_{\max}$).

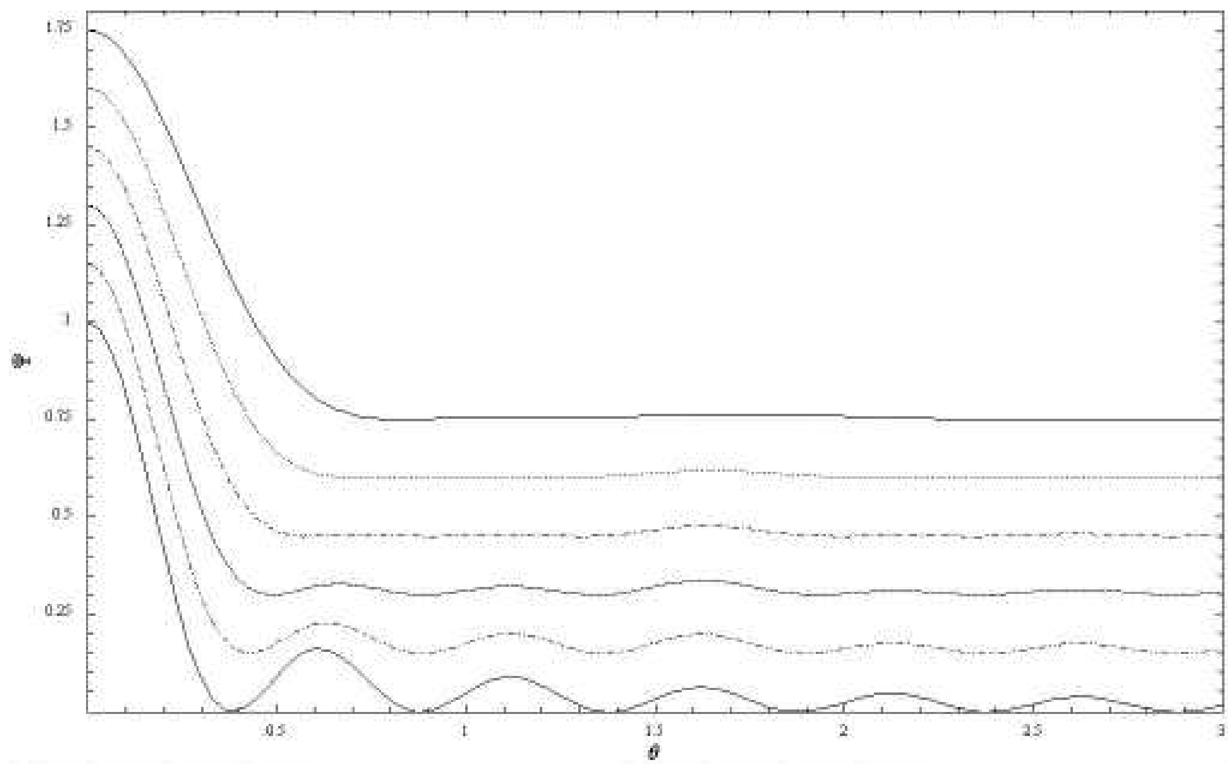


Figure 12: The beam profiles of outer hybrids of type g), with $R = 4$ and no gap (i.e. with $r_{\mathrm{E}} = r_{\mathrm{max}}$).

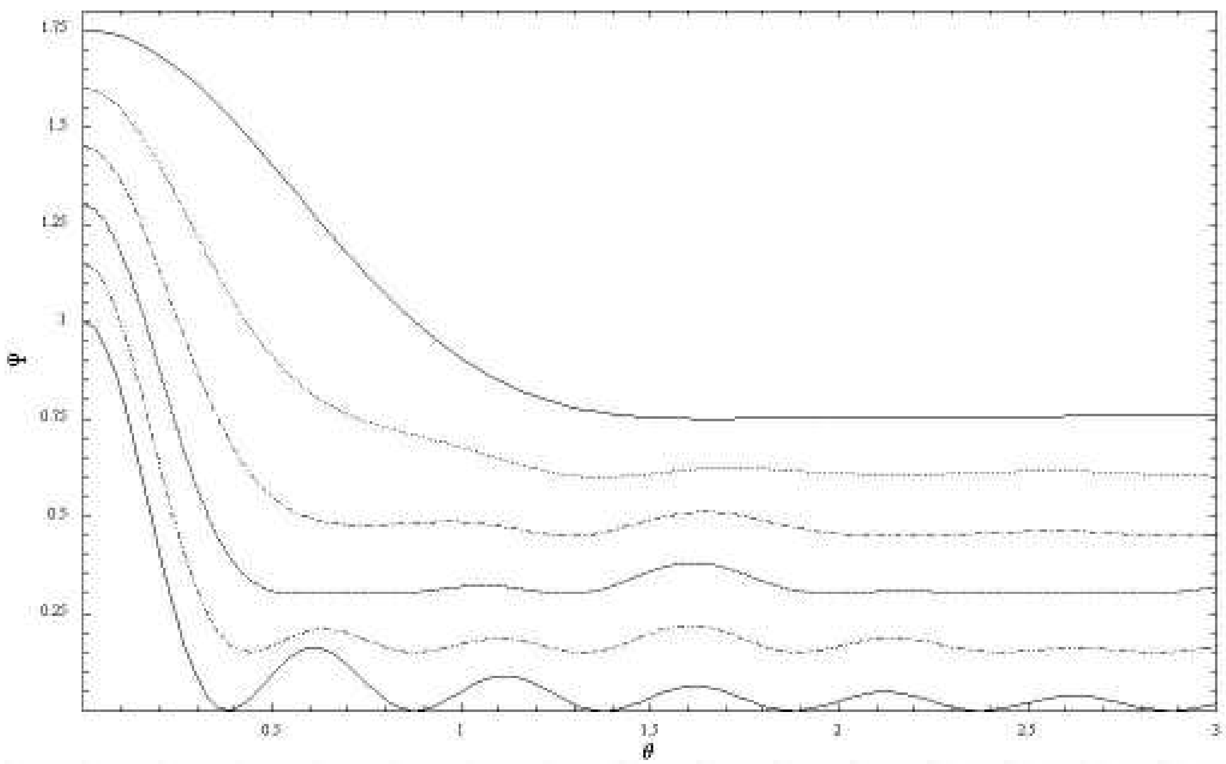


Figure 13: The beam profiles of outer hybrids of type f), with $R = 4$ and a gap of a factor of two (i.e. with $r_{\mathrm{E}} = 2r_{\max}$).

The level of sidelobes, however, behaves differently from that of the inner hybrids. In all the cases investigated the level is low for the intermediate configuration ($f = 0$) but high for the outer ring ($f = 1$), and increases as antennas are transferred from the former to the latter. In all cases the growth of the sidelobes is approximately monotonic in $f$, the quality of the beam degrading gracefully as $f$ is increased. The different cases may be ranked by the quality of the hybrids in which about half of the antennas have been transferred (i.e. those with $f = 0.4$ and 0.6); Figs 10, 12 and 13 show the best quality in this regard.

Since the outer hybrids are large in diameter it is not likely that they will often be used in snapshot mode, from which it follows that the quality of the instantaneous beam profile is not of major importance. All the cases presented here are therefore broadly acceptable in terms of beam quality, and it seems likely that any choice between them will have to be made on other grounds.

The properties of the last case presented above may be of particular interest in providing other grounds of this kind because the existence of the large gap between $r_{\rm max}$ and $r_{\rm E}$ may confer certain advantages, the first stemming from the requirement that the array be designed to fit the topography of the site. If the diameter of the outer array is taken to be small, say 3 km or so, then it should be possible to find a region of the plain of Chajnantor that is large enough to contain the entire array while simultaneously being flat and free of obstacles to the siting and transportation of antennas. If, however, the diameter is taken to be large, say 10 km, then there is probably no such region to be found and the outer ring would have to encompass a large hill, making it impossible to build the intermediate configuration right up to the outer ring. A design for an array with a built-in gap between these two configurations might therefore be ideal. The second advantage is connected with economy because the intermediate configuration is an expensive one, containing as it does half the stations of the array under consideration. The presence of the gap would reduce the overall diameter of the intermediate configuration to half that of one extending all the way to the outer ring, halving at a stroke the cost of all the items of infrastructure that are proportional to the length required, such as the roadways, power cables and optical fibres. These advantages come at no real cost in terms of the quality of the synthesized beams and so this case seems worthy of special attention and further study.

## 3.3 General

The method adopted here is based on various simplifying approximations, the first being the continuum approximation in which the behaviour of an array of a finite number of isolated antennas is represented by a density distribution in the form of a continuous function of position. The approximation is equivalent to taking the limit of a finite model in which the number of dishes is progressively increased without limit, $N_{\mathrm{A}} \to \infty$, and so it may be expected that its accuracy will be greater for arrays with large values of $N_{\mathrm{A}}$ than small. The issue for the MMA/LSA is then whether the likely number of dishes ($N_{\mathrm{A}} \approx 60$) is large enough for the approximation to yield useful results, for which purpose a separate calculation of the beam profiles of discrete models will be required.

The second approximation inherent in the method is the assumption of circular symmetry. A distribution of a finite number of antennas cannot have this as an exact symmetry and so the approximation affects all the configurations and hybrids, but the approximation is probably at its most unrealistic when applied to the outer ring. Modelling the ring as a circle may be acceptably close for a large number of antennas distributed on the circumference of a circle, but such a ring is rather a crude design and more sophisticated versions with the positions of the stations randomized and based on a Reuleaux triangle rather than a circle are known to give superior electromagnetic performance (e.g. Keto 1997). Again, the best way to proceed is to drop the approximation and to investigate a model consisting of a finite number of dishes, but the present pattern of results for the outer hybrids already leads us to conjecture that the beam quality will usually degrade gracefully as $f$ is increased. Since arrays of antennas sited on Reuleaux triangles have somewhat lower sidelobe levels than those sited on circles, the corresponding outer hybrids may also be conjectured to have lower sidelobe levels.

Apart from these approximations there are other assumptions implicit in the analysis that might not obtain in practice. One is the assumption that it is possible to deplete the compact configuration uniformly, but it may not be possible to remove an antenna from the interior given that the antennas are sited very close together in this configuration. If it is not possible then two strategies are possible, the first being to remove a peripheral antenna and then to resite an adjacent internal antenna on that peripheral site, and so on, with a vacancy moving progressively inwards rather like the motion of a hole through a semiconductor lattice until the required antenna has effectively been removed. A second strategy is to deplete the compact array from its periphery, where the antennas are accessible, until $f \approx 0.3 - 0.4$, and then to rearrange the antennas on this configuration uniformly and proceed to deplete them uniformly thereafter; this would be a useful strategy because, although removing antennas from the periphery ultimately leads to inner hybrids with unacceptably high sidelobe levels, it does not do so initially but only when $f \geq 0.4$. This mixed hybrid has another interesting property: the first sidelobe of the compact configuration decreases and disappears more rapidly as a functon of $f$ when the antennas are removed from the periphery than when they are removed uniformly, and so if that sidelobe is regarded as large enough to be worth reducing in the inner hybrids it could be eradicated more quickly by this mixed approach.

Another consideration is concerned with the degrees of freedom available when designing hybrids, the number of which is so large that it might well be possible to find designs that are superior to those considered here. This study has been retricted to the investigation of a relatively small number of degrees of freedom, but even so the number of independent combinations is sufficiently large that it was not reasonable to study them all. Nor is it difficult to think of other degrees of freedom, such as modifying the radial distribution of the density of stations in the intermediate configuration away from the inverse-square law near $r_{\min}$ or $r_{\max}$ with the aim of seeking improvements in the corresponding hybrids. No attempt has been made to study these additional possibilities for the simple reason that a few excellent hybrids have already been found whose properties are of such high quality that it seems more profitable to study them further than to seek superior designs.

## 4 Conclusions

Composite arrays consisting of a close-packed compact configuration, an inverse-square intermediate configuration and an extended outer-ring configuration were investigated to determine whether their hybrids have beams of sufficiently high quality to be useful in astronomy. The demands on the inner hybrids are the most stringent because the sensitivity is high enough for them to be useful in snapshot mode, and low instantaneous sidelobe levels are an important requirement. While it was found that different designs of inner hybrid differ substantially in quality, the best of those considered have excellent properties. The outer hybrids, by contrast, are not required to show low instantaneous sidelobe levels and are found in practice to show little variability in quality as judged by the level of sidelobes: the high quality of the beams of the intermediate configuration is found in all cases to degrade progressively as antennas are transferred to the outer ring. The most important finding for the outer hybrids is that leaving a wide gap between the intermediate and outer configurations does not significantly degrade the performance, suggesting a technique for coping with the topography of the site at Chajnantor whilst simultaneously cutting the cost of the infrastructure of the intermediate configuration by a substantial factor. For convenience and simplicity the hybrids were investigated in the continuum approximation, and the most pressing need is to check that the results hold good for a more realistic discrete model consisting of perhaps 60 antennas on 240 stations.

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