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Suggestions for Revised Definitions of Noise Quantities, Including Quantum Effects

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ABSTRACT

Recent advances in millimeter and submillimeter wavelength receivers and the development of low-noise optical amplifiers focus attention on inconsistencies and ambiguities in the standard definitions of noise quantities and the procedures for measuring them. The difficulty is caused by the zero-point (quantum) noise, hf/2 W/Hz, which is present even at absolute zero temperature, and also by the nonlinear dependence at low temperature of the thermal noise power of a resistor on its physical temperature, as given by the Planck law. Until recently, these effects were insignificant in all but the most exotic experiments, and the familiar Rayleigh-Jeans noise formula P = kT W/Hz, could safely be used in most situations. Now, particularly in low-noise millimeter-wave and photonic devices, the quantum noise is prominent and the nonlinearity of the Planck law can no longer be neglected.

The IEEE Standard Dictionary gives several definitions of the noise temperature of a resistor or a port, which include: (i) the physical temperature of the resistor; and (ii) its available noise power density divided by Boltzmann's constant -- definitions which are incompatible because of the nature of the Planck radiation law. In addition, there is no indication of whether the zero-point noise should be included as part of the noise temperature.

Revised definitions of the common noise quantities are suggested which resolve the shortcomings of the present definitions. The revised definitions have only a small effect on most RF and microwave measurements, but they provide a common, consistent noise terminology from DC to light frequencies.

CONTENTS

Introduction

With noise temperatures of millimeter wave receivers and optical fiber amplifiers now approaching the quantum limit, the effects of quantum noise can no longer be ignored. For many years microwave engineers have felt secure in the belief that the noise power available per Hz of bandwidth from a passive resistor at temperature T was given, according to the Rayleigh-Jeans law, by

$$P^{R\text{-}J} = kT \ . \tag{1}$$

In some unusual circumstances, at high frequencies or low temperatures, the departure from the Rayleigh-Jeans law, as given by the Planck formula, was significant and the following expression was used:

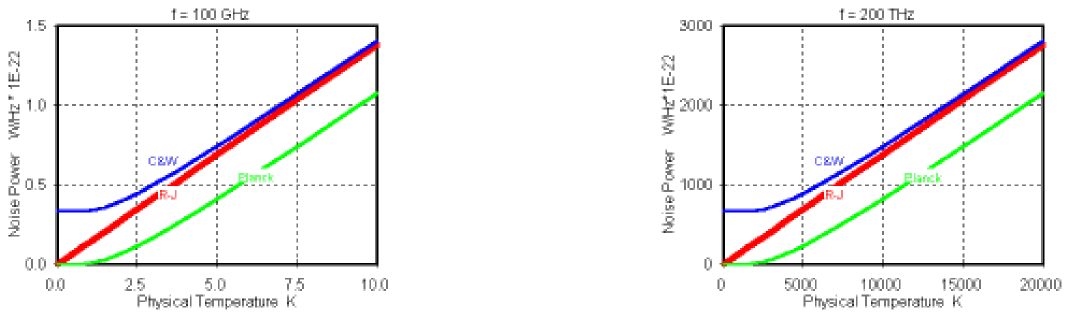
$$P^{Planck} = kT \left[\frac{\frac{hf}{kT}}{\exp\left[\frac{hf}{kT}\right] - 1} \right] . \tag{2}$$

However, there is another noise term which must be included to describe the noise from a passive resistor. It is contributed by the zero-point vacuum fluctuations and has a magnitude hf/2 W/Hz. This term must be added to the Planck formula, giving the result of Callen and Welton [1]:

$$P^{C\&W} = kT \left[\frac{\frac{hf}{kT}}{\exp\left[\frac{hf}{kT}\right] - 1} \right] + \frac{hf}{2} \ . \tag{3}$$

These three equations are plotted in Fig. 1 as functions of temperature for frequencies 100 GHz and 200 THz (the latter corresponding to the 1.5 µm band widely used in photonic communications). It is clear from Fig. 1(a) for 100 GHz, that the Rayleigh-Jeans and Callen & Welton curves are very close for temperatures above ~5 K*i*(*e.,* hf/k). At 200 THz (Fig. 1(b)), the Rayleigh-Jeans and Callen & Welton curves are close only at temperatures above ~10,000 K; below ~3000 K the zero-point noise dominates.

The standard definitions of noise quantities such as noise figure and noise temperature were conceived in the context of components and systems operating at physical temperatures much greater than hf/k and with noise temperatures much greater than the quantum limit, conditions under which the Rayleigh-Jeans law could meaningfully be used. Now, the same noise quantities are being used increasingly in situations in which hf/k is not small compared with the ambient temperature and for systems whose noise temperatures approach the quantum limit. Under these conditions the nonlinearity of the Planck law and the presence of the zero-point noise are important and cannot be ignored in defining and measuring noise quantities. The purpose of this paper is to show that the current definitions of common noise quantities are then inconsistent and ambiguous, and to suggest improved definitions which are compatible with the physics of thermal radiation while disrupting established engineering practice as little as possible.



(a) (b)

Fig. 1. Noise power density vs physical temperature for a resistor, computed at (a) 100 GHz and (b) 200 THz (= 1.5 µm) according to the Rayleigh-Jeans, Planck, and Callen & Welton laws.

Nature of the Zero-Point Fluctuations

Any lingering doubts about the physical reality of the zero-point fluctuations were removed by the recent measurement [2] of the Casimir force, the force on a conductor due to the radiation pressure of the zero-point fluctuations. There has been some uncertainty as to whether the zero point fluctuation noise should be considered part of the noise of a resistor or whether it should rather be associated with the electromagnetic mode that connects to the resistor's terminals, or even with the amplifier used to measure the noise. Some authors believe that the zero-point fluctuations should be excluded from consideration of noise powers because they do not represent *exchangeable* power (it being impossible to extract net power from a resistor at absolute zero). The view of Devyatov et al. [3] is that, although the zero-point fluctuations deliver no *net* power, an amplifier with a resistor at its input nevertheless "...develops these quantum fluctuations to quite measurable fluctuations..." at its output. At the receiver input "...one can imagine two zero-point fluctuation waves propagating in opposite directions..." with no net power flow. The zero-point fluctuations, they argue, should be associated with the incoming radiation and not with the receiver itself. Caves, in his definitive paper on quantum noise [4], refers to "...the zero-point noise associated with the input signal...". This view is consistent, too, with Tucker's quantum theory of tunnel-junction mixers [5] which predicts a minimum shot noise contribution exactly equal to that required to satisfy the Heisenberg uncertainty principle *if* the zero-point noise is considered part of the incoming signal.

Practical Consequences

In making a measurement of the noise of a microwave receiver using the standard Y-factor method, the noise power radiated by hot and cold loads is usually assumed to be proportional to the physical temperature of the loads -- *i.e.*, according to the Rayleigh-Jeans law. We now examine the error introduced by this approximation.

In a Y-factor measurement, the hot and cold noise sources are connected individually to the receiver input, and the ratio, Y, of the receiver output powers is measured. From the Y-factor, the intrinsic noise of the receiver can be deduced and expressed as an equivalent input noise power density, an equivalent input noise temperature, or a noise figure. While noise temperatures and noise figures are most commonly used, the discussion will be clearer if we consider noise powers initially.

Let P_R be the equivalent input noise power of the receiver in a bandwidth B, the measurement bandwidth. B is defined by a bandpass filter at the receiver output (for a coherent receiver (*e.g.*, amplifier or mixer) an input filter is unnecessary). With a power P_{in} incident on the receiver in bandwidth B, the measured output power of the receiver $P_{out} = G(P_R + P_{in})$, where G is the gain of the receiver. With hot and cold loads in front of the receiver the measured Y-factor is:

$$Y = \frac{P_R + P_{hot}}{P_R + P_{cold}} \quad . \tag{4}$$

The equivalent input noise power is found by inverting this equation:

$$P_R = \frac{P_{hot} - Y\,P_{cold}}{Y - 1} \quad . \tag{5}$$

The hot and cold loads are simply black-body radiators (well matched waveguide or free-space loads) heated or cooled to accurately known physical temperatures T_{hot} and T_{cold}. Knowing these temperatures, the powers P_{hot} and P_{cold} are deduced from one of the equations (1)-(3).

If the Rayleigh-Jeans equation (1) is used in (5), the resulting equivalent input noise power density of the receiver is:

$$P_R^{RJ} = k\,\frac{T_{hot} - Y\,T_{cold}}{Y - 1} \quad . \tag{6}$$

The Planck equation (2) gives:

$$P_R^{Planck} = \frac{P_{hot}^{Planck} - Y\,P_{cold}^{Planck}}{Y - 1} \quad , \tag{7}$$

and the Callen & Welton equation (3) gives:

$$P_R^{C\&W} = \frac{P_{hot}^{C\&W} - Y\,P_{cold}^{C\&W}}{Y - 1} \quad . \tag{8}$$

Using the result from (2) and (3) that $P^{C\&W} = P^{Planck} + \frac{hf}{2}$, it follows that the equivalent input noise powers, P_R^{Planck} and $P_R^{C\&W}$ of the receiver also differ by hf/2:

$$P_R^{C\&W} = P_R^{Planck} - \frac{hf}{2} \ . \tag{9}$$

Using the Callen & Welton formula for the power radiated by a black body gives an equivalent receiver input noise power lower by hfB/2 (half a photon) than when the Planck expression is used. When the Callen & Welton formula is used, the power radiated by the hot and cold loads includes the half photon of zero-point noise, whereas with the Planck formula the zero-point power is not included in the power from the hot & cold loads, and must therefore be assigned to the receiver under test itself.

Noise Temperature

The IEEE Standard Dictionary of Electrical and Electronics Terms [6] gives the following definitions of the noise temperature at a pair of terminals or at a port, at a given frequency:

Def. 1 *...the temperature of a passive system having an available noise power per unit bandwidth equal to that of the actual terminals.*

Def. 5 *...a temperature given by the exchangeable noise power density divided by Boltzmann's constant...*

These two definitions give different numerical results for a given source of noise. In addition, the meaning of *available* (or *exchangeable*) *noise power* is not clear: does it include the half photon of zero-point noise, or not?

With the first definition -- noise temperature equals physical temperature -- it is clear from Fig. 1 that noise power is not simply proportional to noise temperature, which is inconvenient in many situations; for example, where two (uncorrelated) noise sources are combined through a directional coupler the resulting noise temperature would not be the sum of the separately coupled noise temperatures.

In the second definition, noise temperature is simply a measure of noise power per Hz. This is the usage familiar to microwave engineers. It is also used by most millimeter and submillimeter receiver engineers, and results in straightforward noise temperature calculations.

Of course, the confusion could be completely avoided by refraining from using noise temperatures at all and always using noise power densities, thus a 290 K noise temperature would (at low frequencies) be referred to as 4.002×10^{-21} W/Hz, but it is too late to ban the use of noise temperatures, and besides, in the Rayleigh-Jeans regime there is the convenient equivalence between physical temperatures and noise temperatures to which we are all accustomed.

For the rest of this paper the second definition of noise temperature (IEEE def. 5) will be assumed unless otherwise stated: (noise temperature) = (noise power per Hz)/k. Equations (2) and (3) give the power available from a resistor at temperature T according to the Planck and Callen & Welton laws, and they can now be expressed as noise temperatures as follows:

$$T^{R-J} = T \ , \tag{10}$$

$$T^{Planck} = T\left[\frac{\frac{hf}{kT}}{\exp\left[\frac{hf}{kT}\right]-1}\right], \tag{11}$$

and

$$T^{C\&W} = T\left[\frac{\frac{hf}{kT}}{\exp\left[\frac{hf}{kT}\right]-1}\right] + \frac{hf}{2k}. \tag{12}$$

Equations (10)-(12) are plotted in Fig. 2 for frequencies 100 GHz and 200 THz.

(a) (b)

Fig. 2. Noise temperature vs physical temperature for a resistor, computed at (a) 100 GHz and (b) 200 THz (= 1.5 µm) according to the Rayleigh-Jeans, Planck, and Callen & Welton laws.

Receiver Noise Temperature

The IEEE Standard Dictionary of Electrical and Electronics Terms [6] does not define *receiver noise temperature*, but two widely used definitions are:

(a) ...the noise temperature of a resistor which, connected to the input of a noiseless but otherwise identical receiver, gives *the same output noise power density as that of the actual receiver connected to a source at absolute zero temperature.*

(b) ...the noise temperature of a resistor which, connected to the input of the receiver, results in an *output noise power density twice that of the same receiver connected to a source at absolute zero temperature.*

These definitions are not in fact equivalent unless the zero-point noise is either negligible (Rayleigh-Jeans limit), or is considered part of the receiver (amplifier) noise and not associated with the source. It seems desirable to define the receiver noise temperature not in terms of the noise temperature of a resistor, but rather in terms of the equivalent input noise power density, hence: (receiver noise temperature) = (equivalent input noise power per Hz)/k. The noise temperature measured on a receiver or amplifier using hot and cold loads is then obtained from equations (6)-(8):

$$T_R^{R\text{-}J} = \frac{T_{hot} - Y\,T_{cold}}{Y - 1} \;, \tag{13}$$

$$T_R^{Planck} = \frac{T_{hot}^{Planck} - Y\,T_{cold}^{Planck}}{Y - 1} \;, \tag{14}$$

$$T_R^{C\&W} = \frac{T_{hot}^{C\&W} - Y\,T_{cold}^{C\&W}}{Y - 1} \;. \tag{15}$$

From equation (9) it is apparent that the difference between T_R^{Planck} and $T_R^{C\&W}$ is just one half photon of receiver noise temperature:

$$T_R^{C\&W} = T_R^{Planck} - \frac{hf}{2k} \;. \tag{16}$$

The half-photon of noise temperature is proportional to frequency. At 100 GHz, hf/2k = 2.4 K, about 10% of the noise temperature of the best SIS receivers, and at 200 THz, hf/2k = 4800 K, which is comparable to the noise temperature of the best Er-doped fiber amplifiers.

These three ways (eqns. (13)- (15)) of deducing the noise temperature of a receiver or amplifier from a measured Y-factor are all in current use and all give different answers -- a potential source of confusion at millimeter and shorter wavelengths [7]. It is important to standardize on one of them. By the arguments given in this paper, eq. (15) using the Callen & Welton formula seems the best choice.

Notice that, if definition (1) of noise temperature were used for receiver noise temperature, the results would differ from those in equations (14) and (15). The noise temperature under definition (1) would be the physical temperature of a source having equivalent input noise power density kT_R^{Planck} or $kT_R^{C\&W}$.

Noise Figure

The IEEE Standard Dictionary of Electrical and Electronics Terms [6] gives the following definition of noise figure:

...the ratio of (the total noise power per unit bandwidth delivered by the system into an output termination) to (the portion thereof engendered by the input termination, whose noise temperature is the standard 290 K at all frequencies).

Is the input termination at a physical temperature of 290 K, consistent with IEEE definition (1) of noise temperature, and if so, should its noise power density be obtained using eq. (2) or (3)? Or is its noise power density 290.k W/Hz, consistent with IEEE definition (5)? Other widely used expressions for the noise figure are $F = (S/N)_{in}/(S/N)_{out}$, and $F = 1 + T_R/290$, both of which are subject to the same ambiguities.

The problem with the present IEEE definition of noise figure became apparent to this author on learning that optical fiber amplifiers now achieve noise figures close to 3 dB. This seemed to imply an amplifier noise temperature close to 290 K and an equivalent input noise power density of 290.k, which is far below the minimum, hf/2, required by the Heisenberg uncertainty principle. The explanation of this is that engineers working on photonic devices take the standard 290 K to mean a physical temperature of 290 K [8], as in IEEE definition (1) of noise temperature. At room temperature the noise emitted by a resistor at 200 THz consists almost entirely of its zero-point noise, hf/2k = 4800 K, with an additional thermal (Planck) component of only 4×10^{-11} K.

The ambiguity in the IEEE definition of noise figure is eliminated by modifying the noise figure definition to read:

...the ratio of (the total noise power per unit bandwidth delivered by the system into an output termination) to (the portion thereof engendered by the input termination, whose physical temperature is 290 K).

Then, the minimum possible noise figure consistent with the uncertainty principle in the high frequency/low temperature regime (hf >> kT) is 2 (3.01 dB). In the low frequency/high temperature regime (hf << kT) the minimum possible noise figure is 1 (0 dB).

Such a change of definition modifies the commonly used relation between noise figure and noise temperature: $F = 1 + T_R/290$ becomes $F = 1 + T_R/T_0$, where T_0 is the Callen & Welton noise temperature (eq. (11)) evaluated at 290 K and the frequency of interest. The expression $F = (S/N)_{in}/(S/N)_{out}$ is valid with the stipulation that the incident noise must be that from a passive source at a physical temperature of 290 K. For most RF and microwave measurements such a change will have negligible effect, but at optical wavelengths (*e.g.*, 200 THz) it has a major effect and brings the definition of noise figure into line with what already appears to have become the *de facto* standard in photonics [8].

Recommendations

(1) The thermal noise of a resistor should be defined to include the zero-point fluctuation noise hf/2 W/Hz. The Callen & Welton formula, equation (3), is thus preferred over the Rayleigh-Jeans or Planck formulas, eqns. (1) and (2), when the difference is of consequence.

(2) Noise temperature should be defined as (noise power per Hz)/k. Thus, noise temperature is simply a convenient scaling of noise power per unit bandwidth. This is the present IEEE definition 5, discussed above. IEEE definition 1 should be considered obsolete.

(3) The noise temperature (or equivalent input noise power density) of an amplifier or receiver should not include the zero-point fluctuation noise hf/2k (or hf/2), which should rather be associated with the source. Hence the receiver noise temperature (or equivalent input noise power density) would be obtained from a measured Y-factor using eq. (15) (or eq. (8)) above.

(4) The definition of noise figure should be changed to specify a source at *physical* temperature 290 K, as opposed to a source with a noise temperature of 290 K. This has negligible effect on RF and microwave measurements, and is consistent with current practice in the photonics industry.

Conclusion

It is hoped that this paper will initiate discussion leading to revised definitions of noise quantities which eliminate present ambiguities and inconsistencies. The definitions suggested here would require minimal change in current engineering procedures from RF to optical frequencies, and in very few cases would actually result in a significant change in measured noise parameters or necessitate a change in specifications of components or systems.

The discussion above is not intended to be exhaustive; for example, the revised definitions need to be expanded to cover heterodyne systems, which respond to multiple input frequencies, and consistent definitions of double- and single-sideband noise quantities [7] must be given.

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1. The National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.