MMA Memo 218:
Level of Negative Side Lobes in an Array Beam.

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Abstract

It is shown that the level of negative side lobes in any array beam (PSF - Point Spread Function) is equal $\frac{1}{N-1}$, where N is number of elements at the array. For the central symmetry arrays all negative side lobes are tangent to the horizontal line at the level $-\frac{1}{N-1}$. For large N positive side lobes are bigger than negative ones (in absolute values). For example, for $N = 36$ (accepted value for MMA now) the value of negative side lobes is 0.0286. *The level of negative side lobes in the natural weighting case does not depend on the array configuration and is determined completely by the number of the elements at the array*

1 The approach to the problem

Let's suppose the vector $\vec{r}_i$ determines the position of the array element at the aperture of the array measured at wavelengths. Then the beam pattern of the array is determined by all nonzero baselines and can be verified by the following equation in the natural weighting case.

$$PSF(\vec{e}) = \frac{1}{N(N-1)} \sum_{k\neq n}^{N} \sum_{n\neq k}^{N} e^{-i2\pi(\vec{r}_k - \vec{r}_n)\cdot\vec{e}} \tag{1}$$

where $\vec{e}$ is the vector of the direction on the sky
N is the number of elements in the array.

Now let's calculate the similar double sum including zero baselines.

$$P(\vec{e}) = \frac{1}{N^2} \sum_{k=1}^{N} \sum_{n=1}^{N} e^{-i2\pi(\vec{r}_k - \vec{r}_n)\cdot\vec{e}} = \frac{1}{N} \sum_{k=1}^{N} e^{-i2\pi\vec{r}_k\cdot\vec{e}} \cdot \frac{1}{N} \sum_{n=1}^{N} e^{i2\pi\vec{r}_n\cdot\vec{e}} = |U(\vec{e})|^2 \tag{2}$$

where $U(\vec{e}) = \frac{1}{N}\sum_{n=1}^{N} e^{i2\pi\vec{r}_n\cdot\vec{e}}$ is a voltage beam pattern.

What is the difference between $PSF(\vec{e})$ and $P(\vec{e})$? The number of summands corresponding to the zero baselines at (2) is equal N and all of them are equal 1.
Therefore

$$PSF(\vec{e}) = \frac{1}{N(N-1)}\left[N^2 P(\vec{e}) - N\right] = \frac{N}{N-1}P(\vec{e}) - \frac{1}{N-1} \tag{3}$$

It is seen from equation (2) that $P(\vec{e}) \geq 0$ elsewhere. All minimums of the $P(\vec{e})$ are equal zero. Thus, as it follows from equation (3), all minimums of the $PSF(\vec{e})$ are equal $-\frac{1}{N-1}$. For a central symmetry array, function $U(\vec{e})$ is real and continuous. So it intersects zero line going from its positive lobes to negative ones. Therefore, the minimums of the function $P(\vec{e}) = |U(\vec{e})|^2$ are equal zero and all negative side lobes of $PSF(\vec{e})$ are tangent to the horizontal line at the level $-\frac{1}{N-1}$. Let's illustrate this statement by the one dimension homogeneous array of $N = 10$ and $N = 36$ elements with spacing d.

The functions $P(d \sin(\theta)) = \left[\frac{\sin(N\pi d \sin(\theta))}{N \sin(\pi d \sin(\theta))}\right]^2$ and $PSF(d \sin(\theta))$ are shown at figures (1) and (2) respectively.

The confirmation of the statement has been found in the [1], where the negative side lobes for $N = 36$ and $N = 63$ were found very close to $\frac{1}{N-1}$

2 Conclusion

The level of negative side lobes in the natural weighting case does not depend on the array configuration and is determined completely by the number of the elements at the array

References

[1] J. Conway, MMA memo 216, 1998

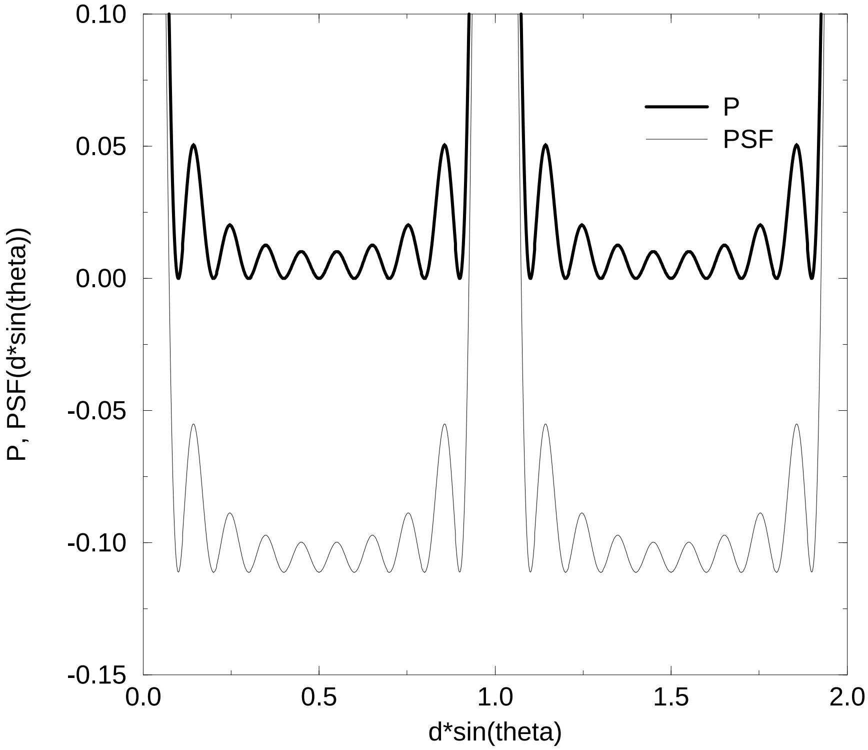

Figure 1: Point spread function PSF($d * \sin(\theta)$) and P($d * \sin(\theta)$) for one dimension homogeneous array of $N = 10$ elemments with spacing d

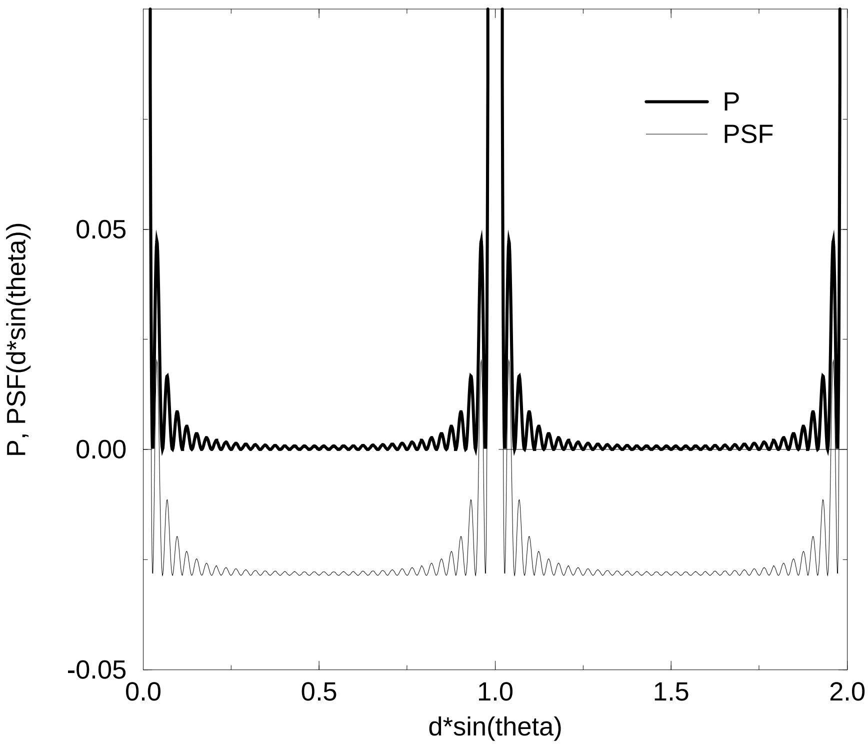

Figure 2: Point spread function $\mathrm{PSF}(d * \sin(\theta))$ and $\mathrm{P}(d * \sin(\theta))$ for one dimension homogeneous array of $N = 36$ elements with spacing d