MMA Memo 217
A, B, C, and D configurations in the shape of concentric circles.

L. Kogan[1]

(1) - National Radio Astronomy Observatory, Socorro, New Mexico, USA

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Abstract

Optimization of an array configuration with a donut constraint [2] showed that the elements tend to group near the donut's circumferences. This fact pushed me to investigate the optimization with two circle constraint. The two circle configuration has an obvious advantage with regard to maintenance. To minimize the number of circles for the **A**, **B**, and **C** configurations, I selected the inner circle of the **A** configuration as the outer circle of the **B** configuration, the inner circle of the **B** configuration as outer circle of the **C** configuration. The **D** configuration was optimized only with the constraint of minimum spacing, having its outer circle as inner circle of the **C** configuration. The calculation has been carried out for the **A** configuration size of 3000m, the **B** configuration size of 840m, the **C** configuration size of 240m, and the **D** configuration size of 96m. The minimum spacing was set at the 12.8m and the number of antennas was set at 36. The optimization has been provided inside the circle at the sky with diameter 40 synthesize beams. For all four optimal configurations the worst side lobe was around 0.1 which was close to the optimal value I had obtained without any constraints. For **D** configuration, the optimization of the side lobes at the level ~ 0.1 is at the whole sky, if we take into account the primary beam consideration.

1 The relation between the diameter of the optimization circle on the sky and the primary beam of the antenna.

It is clear that the effect of the sources located far away from the map center are weakened by the primary beam of the antenna. Let's denote radius of the circle at the sky where the side lobes of the synthesized array are optimized as R_{opt}. Let's consider the achieved side lobes level is α. Denote the two side width of the antenna primary beam at the level α as θ. Also, assume that the two side width of the cleaning map is $\leq \theta$. If

$$R_{opt} \geq \theta \tag{1}$$

then the effect of the side lobes of any source is diminished at least to the level α. Indeed, if the source is located inside the map (closer than R_{opt} to the center of the map), then all its side lobes located inside the map are less than α, because the distance from the source is less than θ and therefore less than R_{opt}. If the source is located outside the map its effect is diminished at least to the level α by the primary beam.

Let's estimate the antenna's primary beam width. Assume that the voltage illumination of the antenna aperture $U(r)$ is circular symmetric and is described by the following expression:

$$U(r) = \left(1 - \left(\frac{2r}{d}\right)^2\right)^n + b \tag{2}$$

where d is the aperture diameter;
b determines the illumination at the aperture edge;
n is the parameter describing the behavior of the $U(r)$ at the domain $(b, 1)$

Such an illumination provides the following voltage beam shape of the antenna [3]:

$$F(u) = \left(\frac{1}{b + \frac{1}{n+1}}\right) \cdot \left(b\Lambda_1(u) + \frac{1}{n+1}\Lambda_{n+1}(u)\right) \tag{3}$$

where $\Lambda_i(x) = i!\left(\frac{2}{x}\right)^i J_i(x)$; $J_i(x)$ is the Bessel function of order i;
$u = \pi\frac{d}{\lambda}\sin(\theta)$;

Peter Napier told me that the future MMA antenna will have pedestal in the illumination at the level 15 db, which makes the parameter b in the equation (2) equal 0.216. Using this information I estimated the two side width of the antenna primary beam $\theta_{0.1}$ for level ≤ 0.1. This range of levels corresponds to the expected side lobes of the optimized array configurations. For $n = 1$ and $n = 2$ it appeared to be equal:

$$\theta_{0.1} \simeq 2\frac{\lambda}{d} \tag{4}$$

The required radius of optimization can be expressed in the numbers of elements of resolution: $\frac{\lambda}{D}$.

$$R_{opt} = N_{opt}\frac{\lambda}{D} \tag{5}$$

Combining equation (4) with condition (1), the following condition for N_{opt} can be obtained.

$$N_{opt} \geq 2\frac{D}{d} \tag{6}$$

The sizes of **A, B, C, and D** configurations are given currently $D_A = 3000m$, $D_B = 840m$, $D_C = 240m$, $D_D \leq 95m$ (Mark Holdaway's information). Assuming that the diameter of the array antenna $d = 10m$, the required radius of optimization in the numbers of elements of resolution is (equation (6)) $N_{opt} = 600, 168, 48, 20$ for the **A, B, C,** and **D** configurations respectively. My experience with the side lobes optimization shows that it is difficult to obtain a good minimization of the side lobes if $N_{opt} > 50$. Therefore, the effect of the side lobes can be minimized in the whole sky only for the **C** and **D** configurations. For **A** and **B** configurations it can be done only inside the limit area at the sky.

2 Optimization of the side lobes. A, B, and C configuration.

I have set $N_{opt} = 20$, although it is a subject of further discussion. Optimization of an array configuration with the donut constraint [2] showed that the elements tend to group near the donut's two circumference. This fact motivated me to investigate the optimization with the two circle constraint. The relevant enhancement of the software was carried out. The two circle configuration has an obvious advantage with regard of maintenance. To minimize the number of circles for the **A, B,** and **C** configurations, I selected the inner circle of the **A** configuration as outer circle of the **B** configuration; the inner circle of the **B** configuration as outer circle of the **C** configuration. The initial configuration consisted of the evenly distributed elements at the given two circles. The number of elements on each circle was proportional to circle's radius. Such an arrangement provides equal spacing on both circles at the initial iteration. For the all three configurations optimum worst side lobe is around 0.1 which is close to the optimum I had without any constraint [2]. The found configurations (labeled by the diamonds) and the UV coverage (labeled by the dots) are shown on figures (1 2 3)

3 Optimization of the side lobes. D configuration.

I have set $N_{opt} = 20$ which, as shown above, provides the optimization of the side lobes at the level ~ 0.1 at the whole sky, if we take into account the primary beam consideration. The initial configuration consisted of evenly distributed elements on the three circles. The number of the elements on each circle was proportional to each circle's radius. The algorithm of the side lobes optimization with minimum spacing constraint requires the initial configuration to satisfy the condition of the minimum spacing. The required minimum spacing was selected 12.8m (Mark Holdaway information). Using the three circles as the initial configuration, we need to satisfy condition of minimum spacing on the circles and between them. Let's denote the spacing as ΔD and the array's diameter (the outer circle) as D.

Condition of minimum spacing between the three circles.

The condition of equal spacing between the three circles infers that radius of the inner circle is $r_1 = \frac{D}{2} - 2\Delta D$. This radius should be larger than $\frac{\Delta D}{2}$. Combining these two statements we obtain the following inequality:

$$D > 5\Delta D \tag{7}$$

Condition of minimum spacing on the three circles.

The total length of the three circumferences should be longer than $N\Delta D$, where N is number of the elements in the array. Considering equal spacing between the circles, the total length of the three circumference is $3\pi(D - 2\Delta D)$. Combining these two statements we obtain the following inequality:

$$D > \left(\frac{N}{3\pi} + 2\right)\Delta D \tag{8}$$

The both conditions (7), and (8) should be satisfied. Therefore the minimum array's diameter can be found as:

$$D > \max\left(5\Delta D,\ \left(\frac{N}{3\pi} + 2\right)\Delta D\right) \tag{9}$$

The number of the array's elements is set at $N = 36$, the minimum array's diameter is determined as $D > 5.82\Delta D$. For $\Delta D = 12.8m$, the minimum possible array's diameter is $\sim 75m$. I applied the side lobe optimization for several array's diameter starting with $78m$. The result is shown at the table (1).

Table 1: The worst side lobe of the D array versus its diameter D. The diameter of circle area of optimization at the sky $\mathbf{40\frac{\lambda}{D}}$

The array size, m	78	82	86	90	96
The worst side lobe	0.390	0.283	0.125	0.108	0.102

Minimum spacing 12.8 meters.

We see from the table that the optimization of the side lobes gives a good result starting with the array's size $D = 86m$. For $D \geq 90m$ the optimum worst side lobe is practically equal to the optimum I had without any constraint [2]. The found configuration for $D = 96m$ (labeled by the diamonds) and the

UV coverage (labeled by the dots) are shown on figure (4). Just note that the side lobes of the initial configuration without optimization is around 0.5-0.6 depending on the array's size.

4 Conclusion

The two circle array configuration has been considered for **A, B,** and **C** configuration. To minimize the number of the circles, the inner circle of the **A** configuration is used as the outer circle of the **B** configuration; the inner circle of the **B** configuration is used as the outer circle of the **C** configuration. The **D** configuration is optimized inside the circle diameter 96 meters with the minimum spacing being 12.8 meters. The achieved minimum worst side lobe is ~ 0.1 for the proposed **A, B, C,** and **D** configurations. The result practically coincides with the minimum obtained after optimization without any constraint. Figures (5), (6), and (7) show the found optimum configurations at different scales in meters.

References

[1] L.R. Kogan, MMA memo 171, 1997

[2] L.R. Kogan, MMA memo 212, 1998

[3] The Handbook of Antenna Design, vI, editors A.W. Rudge, K. Milne, A.D. Olver, and P. Knight, Peter Peregrinus Ltd., London, 1982

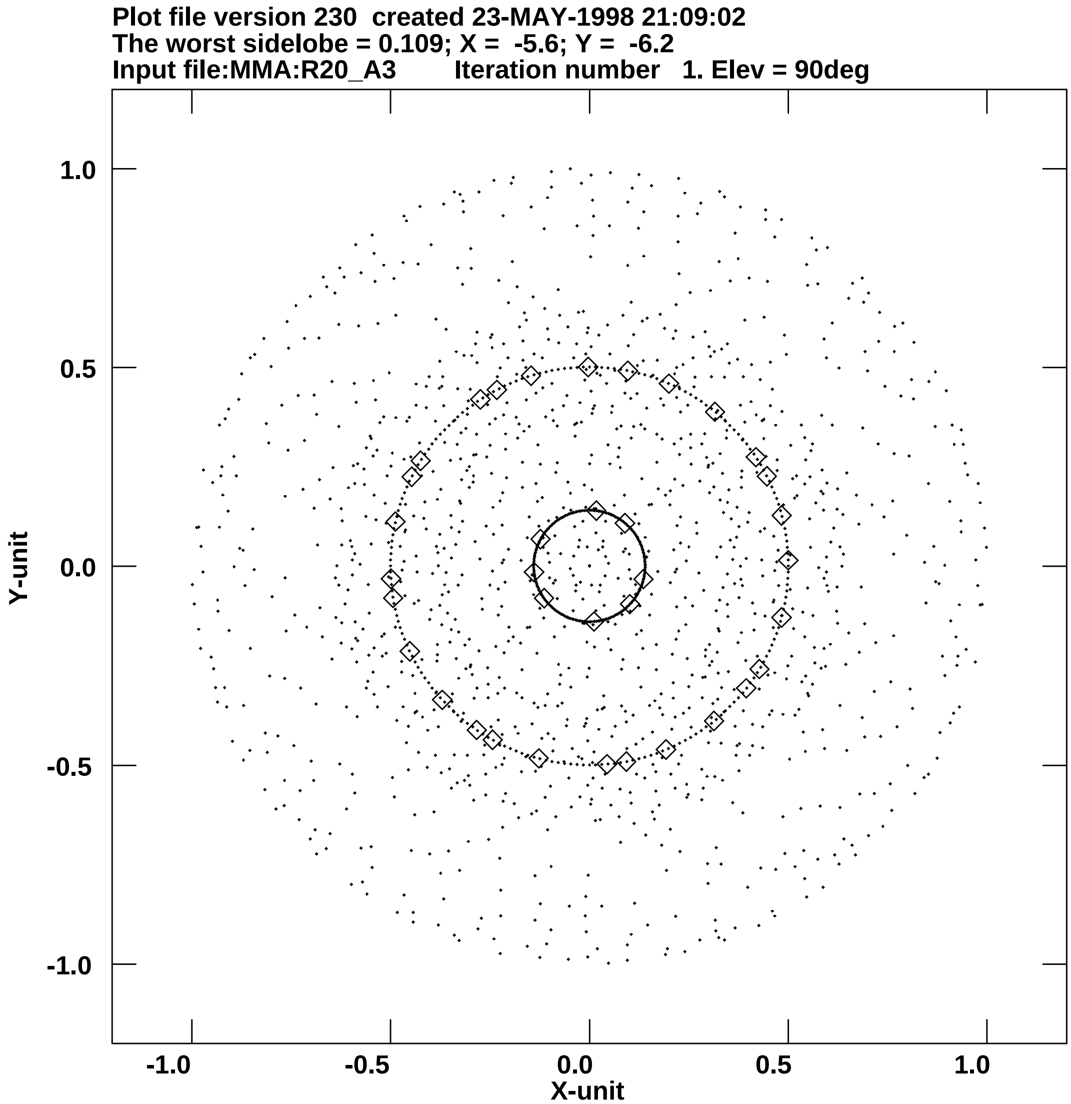

Figure 1: The optimum **A** configuration (diamonds) and UV coverage (dots). The outer diameter is 3000 m. The inner diameter is 840 m. Diameter of the circle of optimization at the sky is $\mathbf{40}\frac{\lambda}{\mathbf{D}}$.

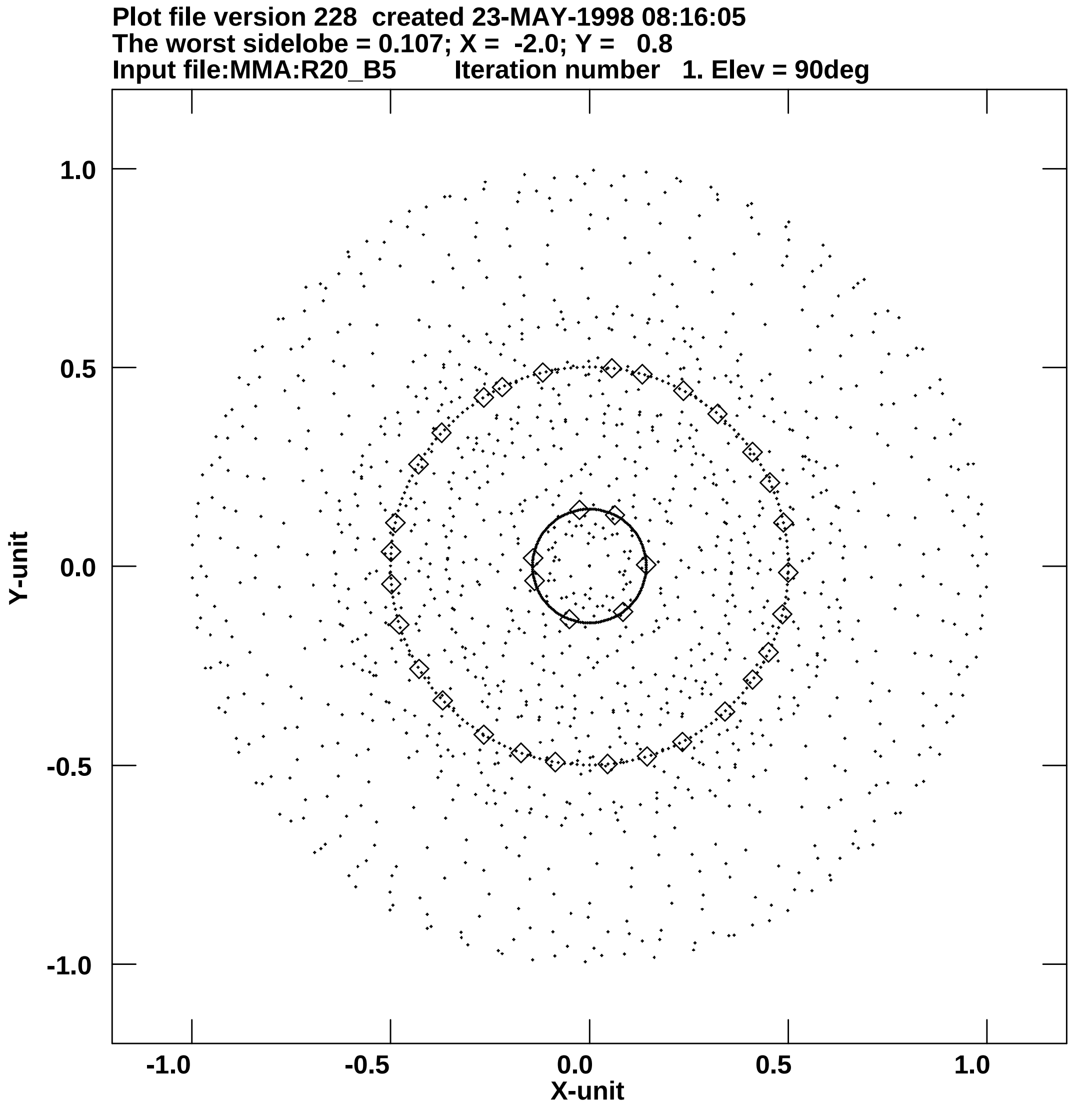

Figure 2: The optimum **B** configuration (diamonds) and UV coverage (dots). The outer diameter is 840 m. The inner diameter is 240 m. Diameter of the circle of optimization at the sky is $\mathbf{40\frac{\lambda}{D}}$.

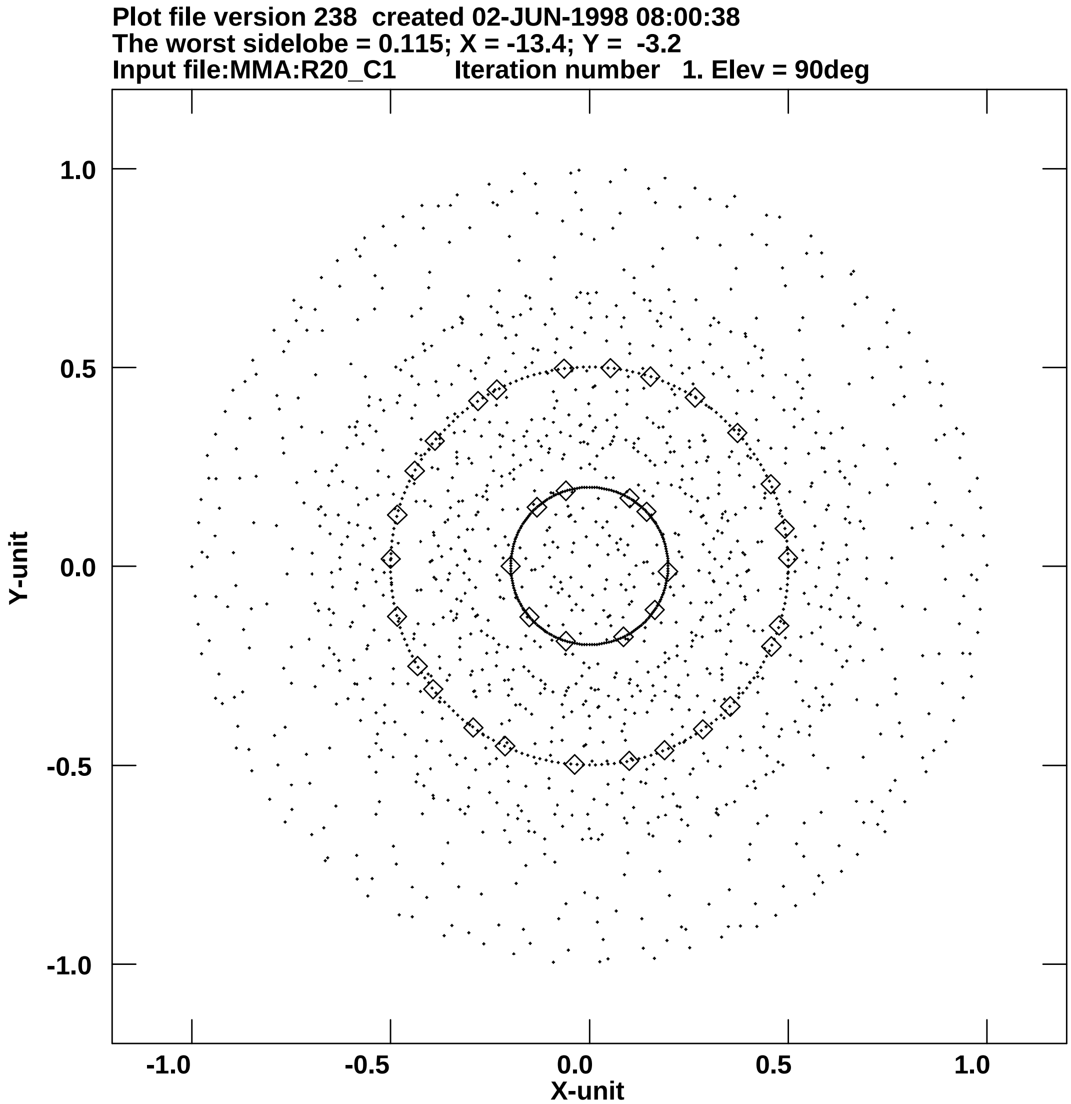

Figure 3: The optimum **C** configuration (diamonds) and UV coverage (dots). The outer diameter is 240 m. The inner diameter is 95 m. Diameter of the circle of optimization at the sky is $\mathbf{40}\frac{\lambda}{\mathbf{D}}$.

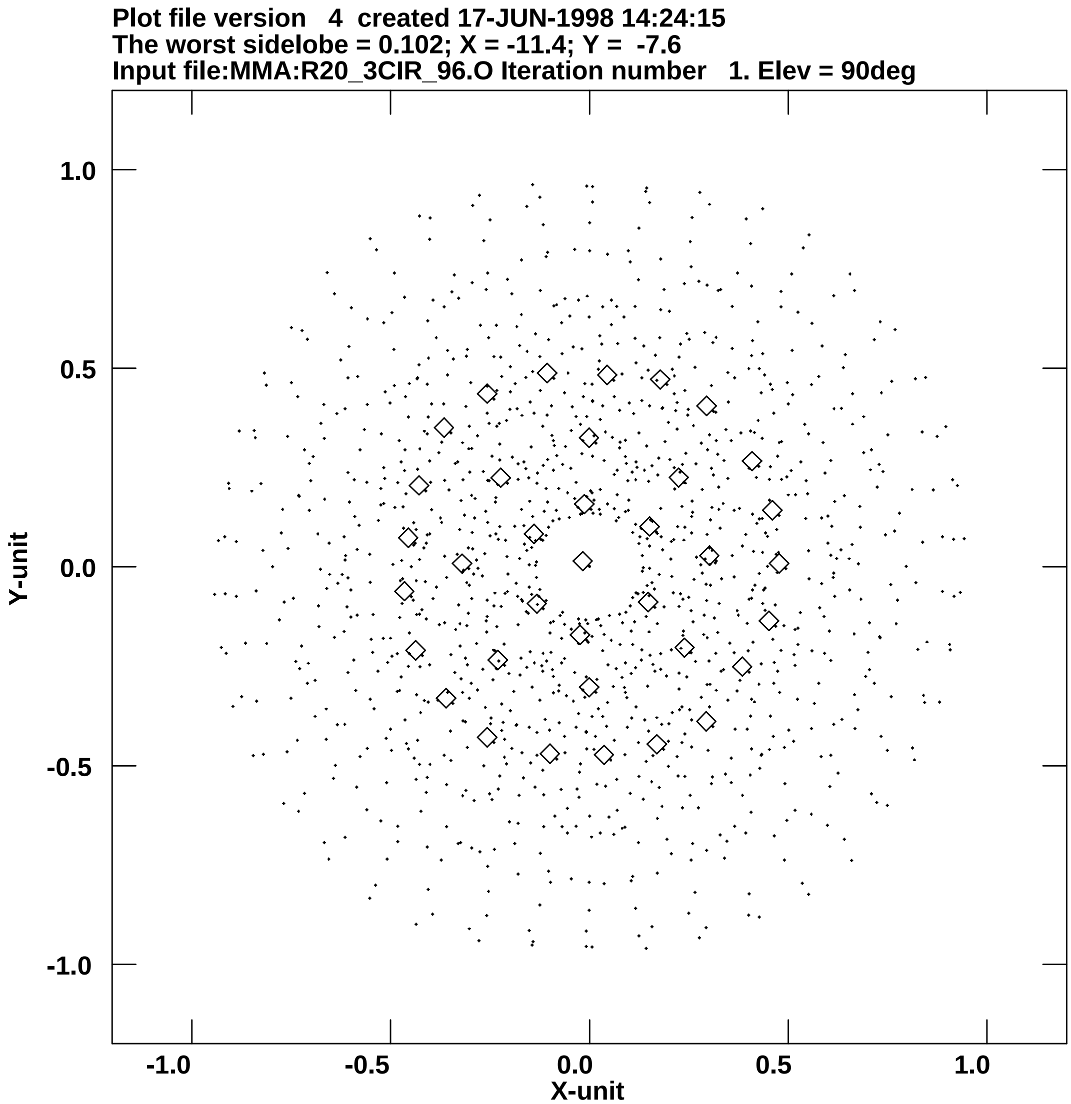

Figure 4: The optimum **D** configuration (diamonds) and UV coverage (dots). The outer diameter is 96 m. Diameter of the circle of optimization at the sky is $\mathbf{40\frac{\lambda}{D}}$. The whole sky optimization, if we include the primary beam consideration.

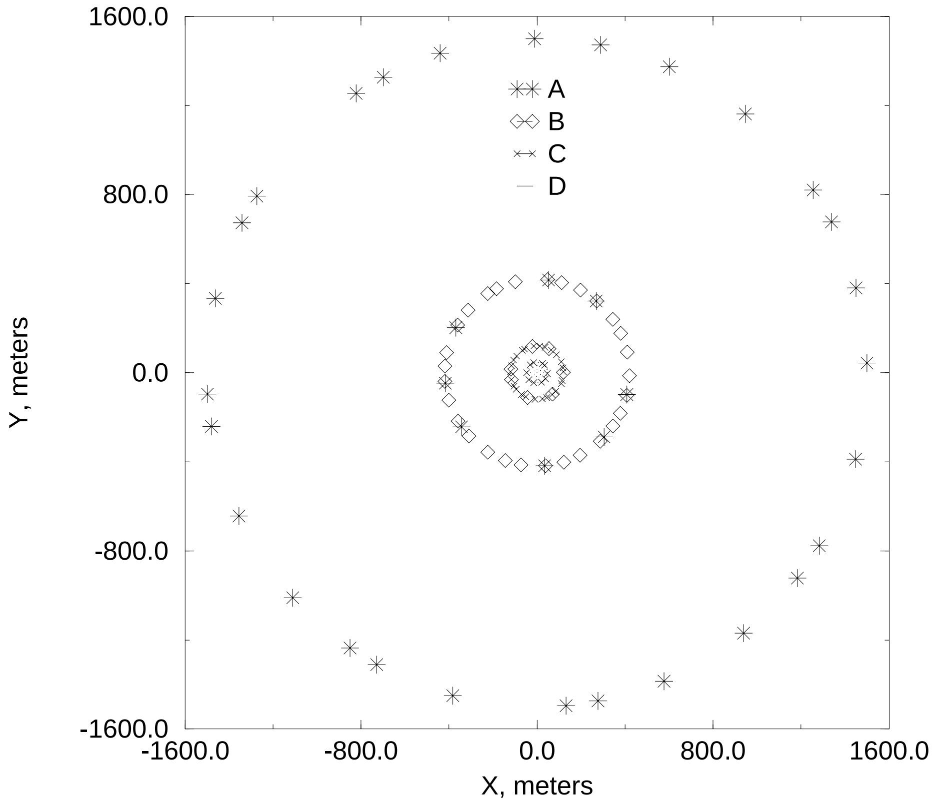

Figure 5: The optimum **A, B, C,** and **D** configurations at the actual scale in meters. Diameter of the circle of optimization at the sky is $\mathbf{40\frac{\lambda}{D}}$.

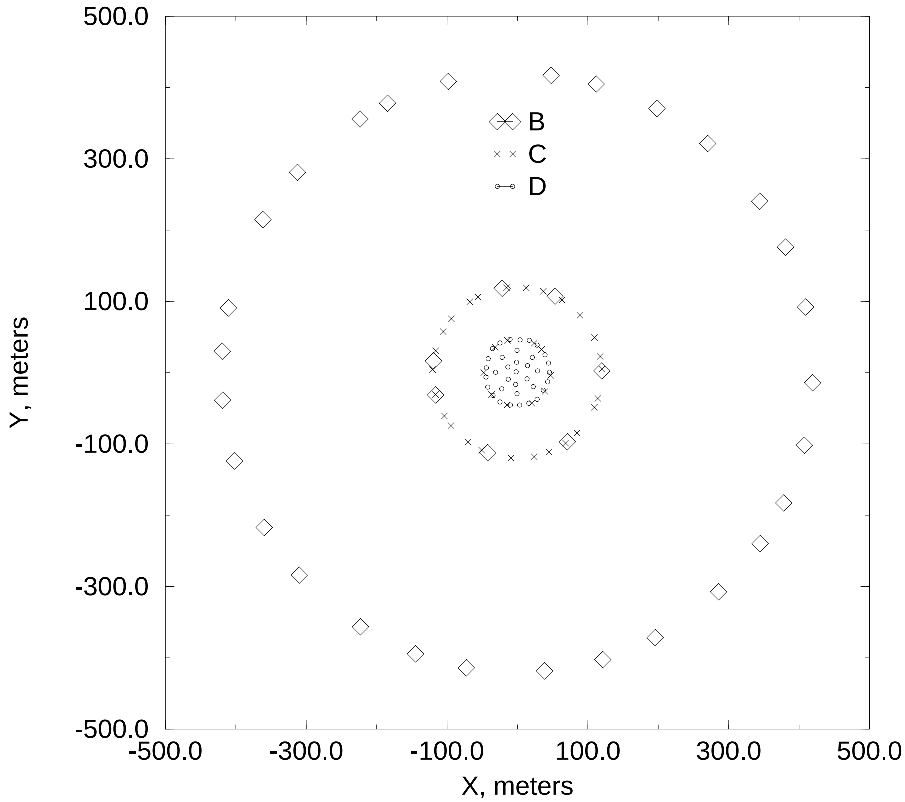

Figure 6: The optimum **B**, **C**, and **D** configurations at the actual scale in meters. Diameter of the circle of optimization at the sky is $\mathbf{40\frac{\lambda}{D}}$.

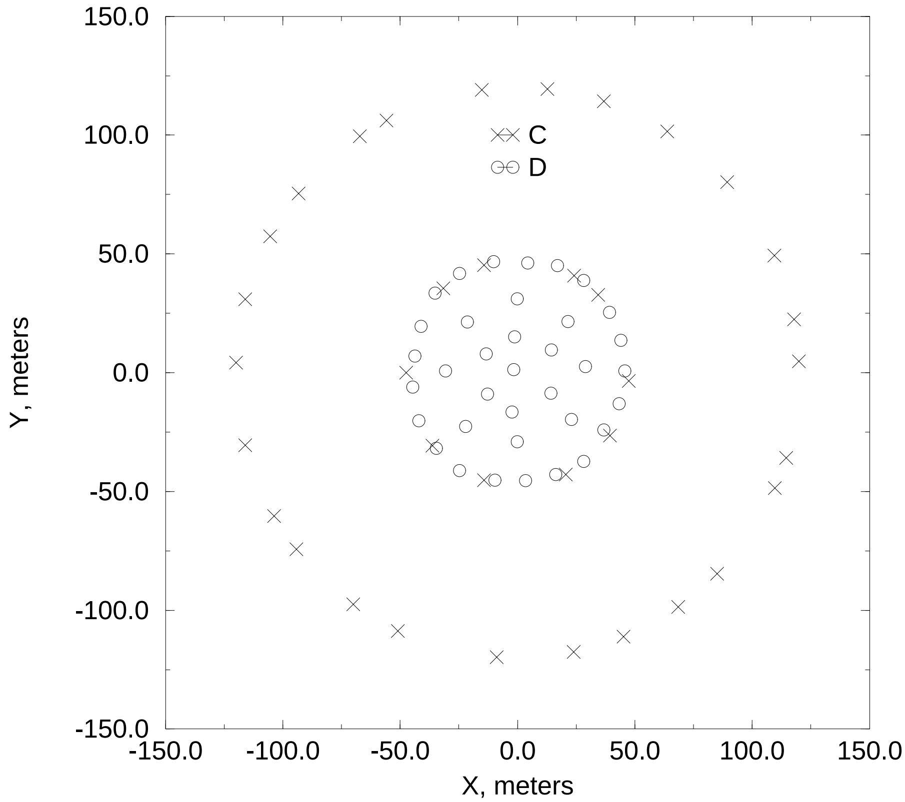

Figure 7: The optimum **C,** and **D** configurations at the actual scale in meters. Diameter of the circle of optimization at the sky is $\mathbf{40\frac{\lambda}{D}}$.