MMA Memo 168: Relative Sensitivities of Single and Double Sideband Receivers for the MMA

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Development of sideband separating SIS mixers (Kerr and Pan 1996, MMA Memo. 151) will allow the use of single sideband receiving systems at frequencies above 100 GHz for the MMA. To observe both sidebands simultaneously with a sideband separating system and retain the full 8 GHz bandwidth would require doubling the IF system and the correlator of the MMA over what has been envisaged up to this point (see MMA Memo. 142). Here, we do not suggest such an expansion but consider the use of just one sideband of a sideband separating mixer at a time, resulting in a single sideband system. With a double sideband system it is possible to observe both sidebands simultaneously and separate the signals in the two bands after correlation. To examine the relative merits of single and double sideband systems for the MMA it is necessary to consider their relative sensitivities. It will be assumed that IF bandwidths and integration times are the same for both systems.

The system noise temperature of a receiving system can be defined as 1/k times the power per unit bandwidth of a noise source at the input of a hypothetical noise-free (but otherwise identical) system that would produce the same noise level at the receiver output, k being Boltzmann's constant. For a double sideband receiver the system noise temperature is described as double sideband or single sideband depending on whether the noise source emits equally in both sidebands or in only one, respectively. With these definitions the single sideband system noise temperature with a double sideband receiver is twice the double sideband noise temperature. To compare the sensitivity of double and single sideband cases it is convenient to introduce a factor

$$\alpha = \frac{double\ sideband\ system\ temp.\ of\ doublesideband\ system}{system\ temperature\ of\ single\ sideband\ system} \tag{1}$$

Then the required relative sensitivities (for an interferometer) are as follows:

Single sideband = 1
Double sideband (continuum, both sidebands used) = $1/(\sqrt{2}\alpha)$
Double sideband (spectral line, one sideband used) = $1/(2\alpha)$

The sensitivity is defined as the modulus of the observed signal divided by the rms noise. In double sideband operation it is assumed that the sidebands are separated after correlation.

With a double sideband system both sidebands may be used, as in a continuum observation, or just one sideband, as in a spectral line observation where the lines of interest occur only in one sideband. In a spectral line observation in which lines of interest occur in both sidebands the observing time is effectively twice that when using one sideband only, so in such cases the sensitivity may be considered equal to that for continuum observation. Another way of comparing the performance of single and double sideband systems is to note that for observations in one sideband one would expect the sensitivities to be equal if the single sideband system temperature of the double sideband system is equal to the system temperature of the single sideband system. In that case $\alpha = 1/2$, so the relative sensitivities given above are consistent with this expectation. [It may also be mentioned that they are consistent with expressions given by Rogers (1976, see Table 1) and by Thompson et. al. (1986, see Table 6.1) which apply to the case of $\alpha = 1$.]

If T_{DS} is the double sideband system temperature[1] of a double sideband system, then

$$T_{DS} = T_C + T_{At} + T_A + T_R \tag{2}$$

where T_C is the cosmic background brightness temperature, T_{At} is the thermal noise from the atmosphere, T_A is the antenna temperature due to sources other than the atmosphere (ground pickup, losses, etc.), and T_R is the double sideband receiver noise temperature. To determine the noise temperature of a radiator at a given physical temperature it is necessary to use the Planck radiation formula[2]. If we take 265 K as the atmospheric temperature, then

$$T_{At} = (1 - e^{-\tau}) \cdot \frac{\frac{h\nu}{k}}{[exp\frac{h\nu}{265k} - 1]} \tag{3}$$

where τ is the atmospheric opacity, ν represents frequency, and h is Planck's constant.

A double sideband mixer receiver can be made into a single sideband receiver by filtering out the unwanted sideband ahead of the mixer, or by using two mixers with quadrature hybrids in the signal and IF connections, as is proposed in the development by Kerr and Pan. In either case the input to the unwanted sideband is usually terminated in a load at the Dewar temperature of approximately 4 K. If T_L is the noise temperature of the load, and T_{SS} is the system temperature of a mixer receiver adapted for single sideband operation as described above, then

$$T_{SS} = T_C + T_{At} + T_A + 2T_R + T_L \tag{4}$$

Note that in this expression the receiver temperature required is the single sideband value which is $2T_R$. If one makes double and single sideband systems using the same types of mixers the value of α becomes, from Eqs. (2) and (4)

[1] In this memorandum system temperatures are referred to the antenna aperture.

[2] The Planck formula is appropriate when the receiver noise temperature is obtained from a Y-factor measurement in which the noise temperatures of the hot and cold loads are determined using the Planck formula. If the noise powers of the hot and cold loads are determined using the formula of Callen and Welton (1951), then the same formula must be used in calculating the effective noise temperature of any other radiator or load that contributes to the system temperature. This is discussed in detail by Kerr et al. (1997, MMA Memo. 161).

$$\alpha = T_{DS}/(T_{DS} + T_R + T_L) \tag{5}$$

Thus if T_{DS} is dominated by the receiver noise and is not much greater than T_R, α can approach 1/2. If T_{DS} is dominated by the atmospheric noise and T_R is small, α approaches 1. To examine these effects for the Chajnantor site we consider two frequencies, 225 GHz and 675 GHz, both of which are in important atmospheric windows. The first and third quartile values of atmospheric opacity at the zenith are 0.03 and 0.08[3] for 225 GHz as given by Holdaway et al. (1996, MMA Memo. 152). For 675 GHz the corresponding values of opacity are 0.53 and 1.72. For the double sideband receiver temperature, figures equal to $2h\nu/k$ and 4hν/k are used. For $2h\nu/k$ the values of T_R are 21 K at 225 GHz and 65 K at 675 GHz, and these values are considered goals. For 4hν/k the values are 43 K and 130 K and these are considered realistic values at this time. For other parameters the following values are used:

T_C = 0.2 (225 GHz), ~0 (675 GHz) (Planck-formula noise temperatures for cosmic background at 2.7 K)

T_A = 5 K (Example of measured value, see Welch et al. (1995, MMA Memo. 143))

T_L = 0.8 K (225 GHz), 0.01 K (675 GHz) (Planck-formula noise temperatures for load at 4 K)

The resulting values of α and of relative sensitivity are given in Table 1 for $T_R = 2h\nu/k$ and Table 2 for T_R = 4hν/k, for zenith path attenuation in both cases. Table 3 gives values for $T_R = 2h\nu/k$ and a ray path at elevation 30°.

The relative sensitivity of the double sideband system (rows 8 and 9 of the tables compared with row 7) is greatest at the lower frequency and for the zenith path, i.e. where the atmospheric attenuation is lowest. The sensitivity of the single sideband system is somewhat higher relative to the double sideband sensitivity in Tables 1 and 3 where the receiver temperatures are lower. For observations in which only one sideband is required the single sideband system clearly offers an advantage (compare rows 7 and 9). Where both sidebands are used, as in continuum operation, there does not seem to be such a strong case for preferring either single or double sideband when the range of conditions in the three tables is considered. When two lines can be observed simultaneously by using the double sideband system one can in most cases do approximately as well by observing each one for half the time with a single sideband system (compare rows 7 and 8). The values on which the sensitivities are based are open to some question, and in particular it remains to be seen what receiver noise temperatures will be achieved with the sideband separation and wide IF bandwidths which are the goals for the SIS mixer development. If the atmosphere is worse than assumed, or if the receiver temperatures are better, the case for single sideband operation is further strengthened.

We wish to thank Jack Welch for drawing our attention for the need to compare sensitivities for the two types of receiver input and Richard Simon for calculating the opacities at 675 GHz.

[3]These values approximate the first and third quartiles for the best months of the year.

Table 1: $T_R = 2h\nu/k$ (DSB), atmospheric attenuation for zenith path.

1	Frequency (GHz)	225	225	675	675
2	Atmos. Opacity	0.03	0.08	0.53	1.72
3	T_{At} (K)	7.7	20	103	205
4	T_{DS} (K) (DSB system)	35	47	173	274
5	T_{SS} (K) (SSB system)	56	68	238	339
6	α	0.61	0.68	0.73	0.81
7	Rel. Sensitivity, SSB	1	1	1	1
8	Rel. Sensitivity, DSB (both sidebands used) $1/(\sqrt{2}\alpha)$	1.15	1.03	0.97	0.87
9	Rel. Sensitivity, DSB (one sideband used) $1/(2\ \alpha)$	0.81	0.73	0.69	0.62

Table 2: $T_R = 4\mathrm{h}\nu/\mathrm{k}$ (DSB), atmospheric attenuation for zenith path.

1	Frequency (GHz)	225	225	675	675
2	Atmos. Opacity	0.03	0.08	0.53	1.72
3	T_{At} (K)	7.7	20	103	205
4	T_{DS} (K) (DSB system)	56	68	238	339
5	T_{SS} (K) (SSB system)	99	112	367	469
6	α	0.56	0.61	0.65	0.72
7	Rel. Sensitivity, SSB	1	1	1	1
8	Rel. Sensitivity, DSB (both sidebands used) $1/(\sqrt{2}\alpha)$	1.25	1.15	1.09	0.98
9	Rel. Sensitivity, DSB (one sideband used) $1/(2\ \alpha)$	0.89	0.82	0.77	0.69

Table 3: $T_R = 2h\nu/k$ (DSB), atmospheric attenuation for elevation angle 30°.

1	Frequency (GHz)	225	225	675	675
2	Atmos. Opacity	0.06	0.16	1.07	3.44
3	T_{At} (K)	15	38	164	241
4	T_{DS} (K) (DSB system)	42	65	233	311
5	T_{SS} (K) (SSB system)	64	87	298	376
6	α	0.66	0.75	0.78	0.83
7	Rel. Sensitivity, SSB	1	1	1	1
8	Rel. Sensitivity, DSB (both sidebands used) $1/(\sqrt{2}\alpha)$	1.07	0.94	0.90	0.85
9	Rel. Sensitivity, DSB (one sideband used) $1/(2\ \alpha)$	0.76	0.67	0.64	0.60

References

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