# Sensitivity of the MMA in Wide Field Imaging

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**Abstract**

The MMA D array used in mosaicing mode will provide a very sensitive and efficient means of imaging very large fields. With integration times per pointing of only 1 s, the MMA D array can achieve surface brightness sensitivity at 115 GHz and 1.3 km/s spectral resolution of 0.19 K at the full $7''.2$ resolution and 0.076 K when tapered to $14''.4$ resolution ($14''.4$ is similar to the resolution obtained with a 50 m single dish at 115 GHz). Observing 1 s per pointing at 230 GHz with 1.3 km/s spectral resolution, sensitivities of 0.25 K at the full $3''.6$ resolution and 0.10 K when tapered to $7''.2$ resolution can be achieved. With these interesting sensitivity levels in only 1 s per pointing, mosaics of thousands of pointings will be desirable. Utilization of this powerful capability of the MMA will require continuous scan (On-The-Fly) mosaicing, smooth synchronous tracking by all antennas to a few arcseconds pointing accuracy, and correlator dump times of 0.3 s or less.

It can be argued that large single dishes equipped with large focal plane arrays should be more capable at imaging such large regions of the sky. When compared at the same resolution, the MMA is several times faster at wide field imaging than a "straw man" large single dish. Much more computer time will be required to process the MMA data than the single dish data, but the MMA images are likely to be of higher quality.

## Introduction

Traditionally, single dish radio telescopes have been used to image diffuse objects such as atomic or molecular clouds in our Galaxy, and radio interferometers have been used to image extragalactic objects, discrete galactic objects, or compact features embedded in the diffuse galactic objects. While it has been widely recognized that multiple interferometric pointings may be required to image objects of modest size (Rand and Kulkarni, 1990; Adler *et al.*, 1992), the prejudice for observing very large objects with single dishes is still widespread in the radio astronomical community. The design of the MMA attempts to remove this traditional distinction between "single dish science" and "interferometer science" since the MMA D array measures all spatial frequencies from zero up to the longest baseline, thereby permitting observations of arbitrarily large sources (Cornwell 1988, Cornwell, Holdaway, and Uson 1993). In this memo, we look at the sensitivity of the MMA D array to illustrate that very large (i.e., thousands of pointings) mosaic observations are both feasible and interesting, and we also demonstrate that the MMA images wide fields faster than a straw man large single dish with a focal plane array.

### Mosaicing Brightness Sensitivity and Implied Mosaic Size

The surface brightness sensitivity of a single pointing interferometric observation will be

$$\Delta T_b = \frac{T_{sys}}{f \eta_a \sqrt{N_{pol} \Delta t \Delta \nu}}, \tag{1}$$

where $f$ is the filling factor of the array, or the fraction of the array area which is filled with antenna collecting area, $\eta_a$ is the aperture efficiency of a single element, $N_{pol}$ is the number of polarizations, $\Delta t$ is the observing time, and $\Delta\nu$ is the bandwidth. For mosaicing, the brightness sensitivity is further improved by the overlap in pointings, which is given by

$$\epsilon_m(\mathbf{x}) = \sqrt{\Sigma_p A(\mathbf{x} - \mathbf{x}_p)^2}, \tag{2}$$

where $A(\mathbf{x})$ is the primary beam, and $\mathbf{x}_p$ is the $p^{th}$ pointing position. For a regular square grid of Nyquist sampled interferometer pointings, $\epsilon_m$ is constant (except at the edge of the sky coverage) and equal to 1.60. Oversampling by a factor of two in each direction would increase $\epsilon_m$ by a factor of about 2, but the amount of time per pointing would decrease by 4, resulting in the same sensitivity. Hence, the mosaicing sensitivity specific to our rectangular Nyquist sampled grid applies to the general case. So, for rectangular grid mosaic observations, the surface brightness sensitivity is

$$\Delta T_b = \frac{T_{sys}}{f \epsilon_m \eta_a \sqrt{N_{pol} \Delta t \Delta \nu}}, \tag{3}$$

with $\Delta t$ now being the time per pointing. For CO(1-0) at 115 GHz, imaged with the D array at full resolution ($7\farcs2$), with $T_{sys} = 100\ K$, $f$ for the D array is 0.5, $\epsilon_m = 1.60$, $\eta_a = 0.7$, $N_{pol}$=2, $\Delta t = 1$ s, and $\Delta\nu = 0.5$ MHz, or 1.3 km/s, resulting in 0.19 K surface brightness sensitivity. However, if we taper the image to half the resolution, the beam is 4 times as large and we lose only 1.6 of the point source sensitivity, resulting in an improvement in surface brightness sensitivity of $4/1.6 = 2.5$, or $\Delta T_b = 0.076$ K. For CO(2-1) at 230 GHz, $T_{sys} = 200\ K$, full spatial resolution ($3\farcs6$ spatial resolution and 1.3 km/s spectral resolution), $\Delta T_b = 0.25$ K; and at half spatial resolution ($7\farcs2$) $\Delta T_b = 0.10$ K.

Past surveys of galactic CO (see e.g. Combes, 1991) have had sensitivities ranging from 0.5 K down to 0.1 K, typically either with telescope beams of $1'$ (which allows observation of only a very small fraction of a percent of the sky), or with telescope beams of about $8'$ (which permits surveying a substantial fraction of the sky with a dedicated instrument). The sensitivity of the MMA in only one second per pointing is comparable to that of the most sensitive of these surveys, but at resolutions of $\sim 7''$. This gives us the possibilities of imaging very large regions of the sky at $\sim 0.1$ K sensitivities (about 0.5% of the sky in 1 month of dedicated observing, far exceeding the paltry fractions of the sky which have been observed in CO at $1'$ resolution in past surveys, typically 0.01% of the sky), and of spending more time per pointing to perform more sensitive surveys of weaker molecular transitions, such as $^{13}$CO.

We can conclude from the sensitivity arguments and comparisons with the sensitivities of past CO surveys that mosaicing with as little as 1 s per interferometric pointing is actually a very interesting way to spend MMA time because it probes new phase space (namely higher spatial resolution) in a time efficient manner. At 1 s per pointing, ~3000 pointings can be made in an hour, resulting in mosaics of 600 x 600 $2''.9$ pixels (full resolution at 115 GHz), about a quarter square degree per hour.

## Sensitivity Comparison Between the MMA and a Large Single Dish

It might seem that a large single dish with a multi-feed array would be a more practical means of mapping such large regions of the sky; in fact, the MMA will be very fast at wide field imaging. To demonstrate the MMA's speed, we compare the imaging speed of the proposed MMA with a "straw man" 50 m single dish with a 32 beam focal plane array. This is an interesting comparison because the two instruments have the same collecting area, and the 32 beam focal plane array is the largest focal plane array currently planned. To calculate the relative sensitivity of these two instruments, we first consider the number of pointings each instrument must make to cover the same large piece of sky, and we then ask what the sensitivity is on the sky given a certain amount of time observing that position. We can then determine the relative speeds of the two instruments.

Since the large single dish will be tapered, one single dish beam width will be about $1.22\lambda/50$ m. The MMA dishes will not be tapered (see below), and the primary beam width will be closer to $\lambda/8$ m. So, the ratio of the number of large single dish primary beams to MMA primary beams in some large region will be

$$\frac{N_{Beams,sd}}{N_{Beams,mma}} = ((50/1.22)/8)^2 = 26.2. \tag{4}$$

However, the large single dish will have 32 feeds, so the ratio of the number of antenna pointings required by the large single dish and the MMA to image the same region will be

$$\frac{N_{pt,sd}}{N_{pt,mma}} = 26.2/32 = 0.82. \tag{5}$$

Next, we consider how sensitive the two instruments are at a single sky position in a given amount of time. The sensitivity in Jy of a single dish is given by the equation

$$\sigma_{sd} = \frac{2kT_{sys}}{\eta A_{sd}\sqrt{N_{pol}\Delta t \Delta\nu}}, \tag{6}$$

where $\eta$ is the antenna efficiency, $A_{sd}$ is the collecting area of the dish, and $N_{pol}$ is the number of polarizations measured. We break up $\eta$ into the *surface efficiency* $\eta_{sf}$, the *taper efficiency*

$\eta_t$, the *switching efficiency* $\eta_{sw}$, and *other* efficiencies $\eta_o$ including spill over and blockage. The sensitivity in Jy of an $N$ element interferometer is given by the equation

$$\sigma_{mma} = \frac{\sqrt{2} k T_{sys}}{\eta \epsilon_m A_{mma} \sqrt{N_{pol} \Delta t \Delta \nu N(N-1)/2}}, \tag{7}$$

$A_{mma}$ is the collecting area of one MMA dish, $N$ is the number of antennas in the array. The difference of $\sqrt{2}$ in the sensitivity equations is due to the requirement that the total power receivers must be switched against a reference load, which is not required of the interferometer's receivers. For a mosaicing interferometer, we break up the efficiency into the *surface efficiency* $\eta_{sf}$, the *taper efficiency* $\eta_t$, and *other* efficiencies $\eta_o$ including spill over and blockage. In addition, we must consider the *mosaicing sensitivity overlap* $\epsilon_m$.

- We assume the two instruments have the same $T_{sys}$, the same bandwidth $\Delta\nu$, and the same time on source $\Delta t$. We assume the two instruments are on sites with similar opacities.
- **Collecting Area** $A$.
  - The 50 m single dish has 1963 m$^2$ collecting area.
  - Each 8 m interferometer dish has 50.27 m$^2$ collecting area.
- **Mosaicing overlap** $\epsilon_m$.
  - In properly sampled single dish observations, the beams on the sky will overlap at about the half power point. However, information (sensitivity) that one pointing has about its neighbor cannot be retrieved, and no sensitivity advantage is gained.
  - We have formulated the interferometer speed problem in terms of the number of Nyquist sampled synthesized beams observed per pointing, plus the sensitivity per synthesized beam. However, the pointings are also Nyquist sampled on the sky, so each pointing has some sensitivity which can be used by neighboring pointings. We could have formulated the problem in terms of "number of equivalent full sensitivity (i.e., at the primary beam center) pointings in a single pointing, plus the beam center sensitivity per synthesized beam. It is easier to calculate the ratio of the number of synthesized beams to the number of pointing positions for a large sky region, and augment the sensitivity by the mosaicing overlap $\epsilon_m$, which is 1.60 for a rectangular grid, as calculated above.
- **Taper Efficiency** $\eta_t$.
  - The single dish loses collecting area due to the underillumination of the aperture, required to reduce ground pickup. A 12 dB taper at the edge of the dish results in a taper efficiency of 0.70.

– The MMA dishes will have a fairly uniform aperture illumination which drops precipitously at the edge of the dish. This is to be accomplished by the use of a lens, and it is expected that each dish will have a taper efficiency of about 0.88. In order to make a valid comparison of the brightness sensitivities of the MMA and the large single dish, we must taper the MMA images to obtain the same resolution from both instruments. The taper can be done in the image plane after deconvolution, or in the $(u, v)$ plane prior to deconvolution. Deconvolution will work better if the taper is performed in the Fourier plane. Sensitivity to point sources will be higher if the smoothing is performed in the image plane, but the sensitivity to resolved structure will be about the same for either case. We therefore consider the taper in the $(u, v)$ plane. The large single dish resolution is effectively $1.22\lambda/D$, which requires a Gaussian taper of about 37 m in the interferometer's $(u, v)$ plane, resulting in a "(u-v) taper efficiency" of the interferometer of 0.69, or 0.61 when combined with the dish illumination taper.

- **Surface efficiency $\eta_{sf}$.** It is difficult to make very large single dishes with very accurate surfaces.

  – A very large single dish operating at millimeter wavelengths will likely have a surface like $\lambda/16$ at 1 mm, and we assume here that the surface is 70 $\mu$. At 230 GHz, the surface efficiency will be 0.63.

  – The interferometric dishes will have a surface accuracy of 25 $\mu$, resulting in a surface efficiency of 0.94 at 230 GHz.

- **Switching Efficiency $\eta_{sw}$.**

  – The single dish will need to switch (i.e., difference two signals taken at different sky positions at the same or at similar times) in order to get a reference zero level and also in order to remove the atmospheric emission. In principle, the reference zero level may be obtained accurately with little overhead, so we do not consider it here. The efficiency lost to switching is a complicated issue and depends upon the observing strategy and the reduction and analysis strategy. The most conservative choice spends half of the observing time on source and half of the observing time off source. $\eta_{sw}$ is then 0.5, $1/\sqrt{2}$ due to spending half of the time on source and $1/\sqrt{2}$ due to adding two equally noisy signals. However, if multiple ON positions are switched against a single OFF position, it is possible to get a higher efficiency. In this situation, the optimal time to spend on the OFF position is $\sqrt{N_{on} t_{on}}$ (Emerson, *private communication*), in which case the noise increases by $1 + 1/\sqrt{N_{on}}$, or the switching efficiency becomes about $1 - 1/\sqrt{N_{on}}$. So, if we observed 100 ON positions for 1 s and switched against a single OFF position observed for 10 s, we would see a noise increase of 1.1, or $\eta_{sw} = 0.91$. The number of ON positions which can be switched against a single OFF position is limited by the time scale of the atmospheric fluctuations, and the ON and OFF positions should be taken within about a minute.

So, 100 ONs per OFF is quite reasonable in the case of spectral line on-the-fly mapping, for which an ON position is defined now as a pixel in the final Nyquist sampled image. However, the noise in such an image will be correlated among all of the pixels which were switched against a single OFF position. Since this is not desirable in some circumstances, more frequent switching and lower efficiencies will sometimes be required. (This applies to the total power measured by the MMA as well as to the large single dish.)

  For the single dish, we will consider the case of on-the-fly spectral line mapping with $\eta_{sw} = 0.9$, or one OFF position for 100 ON positions. Note, however, that continuum mapping would require more OFF time, resulting in a lower switching efficiency.

  - An interferometer does not need to switch, so $\eta_{sw} = 1$ for spectral line or continuum.

- **Number of Polarizations** $N_{pol}$**.** If both polarizations are observed simultaneously, the noise decreases by $\sqrt{2}$.
  - Each receiver in the array of feeds will have only one polarization.
  - The MMA measures both polarizations on each antenna.

- **Other Efficiency Factors.** We assume the two instruments have the same aperture blockage and feed spill over efficiencies.

- **Calibration Overhead.** The single dish will require flux scale and bandpass calibration, and the MMA will require flux scale, bandpass, and phase calibration. In the MMA's compact array, the atmosphere will often be essentially phase stable, so a very small fraction of the time will be spent performing phase calibration. We assume that both instruments will have comparable calibration overhead.

These factors are summarized in Table 1 for the 50 m SD and the MMA.
The ratio of the sensitivity equations gives us

$$\frac{\sigma_{sd}}{\sigma_{mma}} = \frac{\sqrt{2}\eta_t^{mma}\eta_{sf}^{mma}\epsilon_m^{mma}A_{mma}\sqrt{N_{pol}^{mma}N(N-1)/2}}{\eta_t^{sd}\eta_{sf}^{sd}\eta_{sw}^{sd}A_{sd}}. \tag{8}$$

For the parameter values in Table 1,

$$\frac{\sigma_{sd}}{\sigma_{mma}} = 3.3 \tag{9}$$

In order to compare the relative speed of the large single dish and the MMA (*ie*, the relative time required to image the same region of the sky to the same sensitivity), we multiply the relative sensitivity on each pointing, squared, by the relative number of antenna pointings required to image some large region of the sky:

$$\frac{T_{sd}}{T_{mma}} = \left(\frac{\sigma_{sd}}{\sigma_{mma}}\right)^2 \frac{N_{pt,sd}}{N_{pt,mma}}, \tag{10}$$

| | 50 m SD | MMA D Array |
|---|---|---|
| Collecting Area (per element) | 1963 $m^2$ | 50.27 $m^2$ |
| N elements | 1 | 40 |
| N polarizations | 1 | 2 |
| Mosaicing Overlap $\epsilon_m$ | 1.0 | 1.60 |
| Taper Efficiency $\eta_t$ | 0.70 | 0.61 |
| Surface Efficiency $\eta_{sf}$ | 0.63 | 0.94 |
| Switching Eff. $\eta_{sw}$ | 0.90 | 1.0 |
| Other Efficiencies $\eta_o$ | 0.85 | 0.85 |

Table 1: Factors influencing the single pointing sensitivity of the Large Single Dish and the MMA tapered to the same resolution.

or

$$\frac{T_{sd}}{T_{mma}} = 8.5 \tag{11}$$

Hence, it appears that the MMA will be significantly more efficient at imaging very large regions than a large single dish. We can turn the equation backwards and ask: given a single dish with the same collecting area as the MMA, how many feeds must it have to be as fast as the MMA in wide field mapping? It must have 8.5 times more feeds than its assumed 32, or about 270 feeds total to keep up with the MMA. With current technology, an array of 270 feeds on a millimeter wavelength telescope would suffer from severe off-axis effects, resulting in the different feeds having quite different beams.

It is a bit precarious to compare the sensitivities of two instruments which have not yet been built. In addition to uncertainties in the above factors, we have not considered these additional factors:

- **MMA Total Power Measurements.** In order to make valid wide field images, the MMA requires total power measurements, which will be made with the MMA antennas (Cornwell, Holdaway, and Uson, 1993). The fraction of time required for the total power observations will depend upon the scientific objectives of the experiment. By requiring continuity of sensitivity as a function of $(u, v)$ distance between the total power to interferometric data boundary (which occurs near the dish diameter in the $(u, v)$ plane, or about 8 m), and assuming that all interferometric elements participate in measuring the total power, the time-on-source required for the total power measurements is about a quarter of the time-on-source required for the interferometric data (for 40 antennas in a compact configuration with 0.5 filling factor). This *sensitivity equalization* rule of thumb is applicable to projects which are as interested in the extended emission as in the more compact emission, such as measuring the power spectrum of the structure in a molecular cloud. Observations which are primarily interested in bright, small scale features such as filaments and shocks require only that the total power not corrupt these features of interest, and respectable images can be made with a factor of $\sim$10-20 less time spent on total power data than is required to equalize the Fourier plane sensitivity.

In our mosaicing speed analysis above, we have assumed that the MMA will measure the total power at the same time as it measures the interferometric visibilities. In the case of the MMA antennas performing a single OFF for many ONS, which fits efficiently with On-The-Fly mosaicing, it will be possible to measure total power and interferometric data simultaneously and efficiently with the 40 MMA antennas, and the total power sensitivity relative to the interferometer sensitivity will far exceed what is required to produce high quality mosaic images. In this case, no modifications to our speed results will be required.

If a more conservative switching scheme is required to make high quality images and there is a single ON for each OFF, it will probably not be wise to measure total power and interferometric data simultaneously. If beam switching is required to remove the atmosphere (as it will be for continuum observations), it may be difficult to measure total power and interferometric data simultaneously. In these cases, it will be more attractive to dedicate a fraction of the observing time (or a fraction of the antennas) to measure only total power, and the interferometric data is taken with the rest of the observing time (or antennas). If we are as interested in the extended structure as the fine scale structure, we require about the same amount of time on the total power data as on the interferometer data. If we are primarily interested in the small scale structure and don't want to be limited by the short spacing problem on the bright, compact regions, the time dedicated to total power observations will be about 10% of the time spent on interferometer observations.

- **Image Quality.** The MMA images will often be better than the large single dish images due to reduced systematic errors.

- **Phase Stability.** We have assumed in this analysis that phase calibration will be a very small overhead for the MMA. The atmospheric phase stability is usually acceptable on the short baselines of interest here at millimeter wavelengths on the Mauna Kea and Chile sites. However, there will be times when the phase stability is too poor for the MMA compact array to observe without phase correction, or for the 50 m single dish to observe *at all*: conditions which result in poor phase stability for the interferometer will result in anomalous refraction for large single dishes. There are more phase correction options open to interferometers, and the MMA would sometimes be able to observe when the large single dish could not.

- **Antenna Blockage.** We assumed that both instruments have the same blockage in the above calculation. One of the current designs for the MMA antenna has no blockage, but the large single dish will have blockage. Including the blockage differences would increase the MMA's relative speed incrementally.

- $T_{sys}$. We assumed above that $T_{sys}$ would be the same for the MMA and for each of the large single dish's pixels. However, it is difficult to produce a multi-beam system with $T_{sys}$ as low as a single beam system. It should also be pointed out that the 80 single beam receivers on the MMA antennas will not be as sensitive as the most sensitive receivers on a single beam single dish.

- **Other MMA Configurations.** The MMA will not be in its compact array all year round, but even if it is in the D array only one quarter of the year, its productivity will be greater than the large single dish's productivity year round.

- **Computing.** The MMA will require much more computing than a large single dish to produce images (see Holdaway and Foster, 1994). However, the fast linear mosaic algorithm should always work well enough for these very large spectral line mosaics which will not push the algorithm's dynamic range limitations.

## Technical Requirements for the MMA

If the antennas observe in "stop and go" mode, as mosaicing is performed currently, something like 5 s will be lost to the antenna settling down to the good pointing accuracy required for mosaicing, and very large mosaicing becomes less attractive. In order to make full use of the power of the MMA, continuous scan (i.e., On-The-Fly) mosaicing (Holdaway and Foster 1994) will be required. In order for On-The-Fly mosaicing to work, the antennas must scan synchronously. However, since these mosaics will not be very high dynamic range, of order 50:1 or 100:1, the 1 arcsecond pointing is not required, rather, we could probably live with 2-3 arcsecond pointing. Jim Ruff, a VLA antenna engineer, says that the synchronous scanning requirement is not particularly hard to meet, but must be included in the design specifications. With 1 s integration times per Nyquist sampled pointing position, the correlator readout needs to be about 0.3 s. An outstanding problem is the shape of the synthesized beam, which will change across the image unless one of the solutions of Holdaway and Foster (1994) is adopted: flagging $(u, v)$ data which fall outside the $(u, v)$ envelope common to all pointings, restricting the observations to a small hour angle range (thereby limiting the size of the mosaics made on any given day), or scanning over the imaged region very fast several times, which requires a very small correlator dump time. Discussion of which scheme is most appropriate is outside the scope of this memo. Flagging the non-overlapping $(u, v)$ points will degrade the resolution (but will enhance the surface brightness sensitivity). Scanning over the region many times will create extreme data rate problems and may not be technically feasible within the constraints of the MMA budget.

## Conclusions

With only 1 s integration per pointing, the MMA D array gives very respectable surface brightness sensitivity, comparable to the best existing surveys at much lower resolution. In order to take advantage of this sensitivity and the associated capability of imaging very large regions of the sky, the MMA correlator must have a fast dump time and the antennas must be able to point well while scanning. Lots of computation will be required to produce these large images, much more than will be required for a large single dish. The MMA will image wide fields considerably faster than a large single dish with the same collecting area and a reasonable number of feeds; in fact, the single dish's focal plane array will require ~270 feeds in order to image as quickly as the MMA. Hence, any scientific justification for a large millimeter wavelength single dish to image large regions of the sky applies equally well to the MMA. If the technical challenges of fast correlator dump times and synchronous slewing can be met at modest cost, it would probably be very fruitful scientifically to add the fast mosaicing capabilities to the MMA.

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