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Relative Observing Speed of Single Antennas and Packed Arrays

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1. The Problem of Instrumental Observing Speed

There are some scientific problems for which a large single antenna and an array of smaller antennas can be used to make comparable scientific observations. Many of the discussions of the instrument to replace the 300ft that took place Dec. 1-2, 1988 involved comparison of the capability of single dishes and arrays. It therefore is useful to make a quantitative comparision of these two types of instruments. In this comparison we will consider only the question of the on-source integration times needed to attain certain rms noise levels for a beam-switch single antenna compared with a correlated array of N antennas. Limitations due to source confusion will be discussed later.

2. Quantitative Analysis of Observing Speed

In order to make the analysis quantitative we need to define a number of variables. Let

k_B = Boltzmann constant

$\Delta\nu$ = frequency bandwidth

λ = wavelength of observation

N = number of antennas in an array

N_{beams} = number of beams for multi-beamed single antenna

D_1, D_n = Diameter of large single dish and the n-th array antenna

T_1, T_n = System temperature for single dish and n-th array antenna

k_1, k_N = sensitivity equation coefficients for single dish and array

B_N = diameter of packed array of N antennas

$\Omega_1 = (\lambda/D_1)^2$, $\Omega_n = (\lambda/D_n)^2$ beam solid angles for single dish and n-th ant.

$f_N = ND_n^2/B_N^2$ = filling factor for array

$\Omega_B = (\lambda/B_N)^2 = (f_N/N)\cdot(\lambda/D_n)^2$ = synthesized beam solid angle for array

Ω_s = solid angle for a large source greater than antenna beam solid angle

ϵ_1, ϵ_n = efficiency factors for single dish and n-th array antenna (with quantization and other effects included for the latter case)

τ_1, τ_N = integration time per field

τ_1^s, τ_N^s = integration time for observing Ω_s (many fields) and we can then write the sensitivity equation for the rms flux density per beam for a single beam-switched antenna as (cf. Christiansen and Hogbom, 1984)

$$\Delta S_1 = \frac{8 \cdot k_B \cdot T_1}{\varepsilon_1 Z\pi \cdot D_1^2 \cdot \sqrt{(\Delta\nu \cdot \tau_1)}} \tag{1}$$

and the comparable equation for an array of N correlated antennas as

$$\Delta S_N = \frac{8 \cdot k_B \cdot T_n}{\varepsilon_N \cdot \pi \cdot D_n^2 \cdot \sqrt{[\Delta\nu \cdot \tau_n \cdot N(N-1)]}} \tag{2}$$

where we will hereafter use the approximation $N \approx \sqrt{[N(N-1)]}$ to simplify things.

2.1 Surface Brightness Sensitivity

Equations (1)-(2) are appropriate for discussions of point source sensitivity. In order to discuss surface brightness sensitivity we need to divide these equations by $2 \cdot k_B/\lambda^2$ times the appropriate beam solid angle for the single dish (Ω_1) and the array (Ω_B), respectively. Using the simple formulas for these beam solid angles we then have

$$\Delta T_1 = \frac{4 \cdot T_1}{E_1 \cdot \pi \cdot \sqrt{(\Delta\nu \cdot \tau_1)}} \tag{3}$$

for a large single dish and

$$\Delta T_N = \frac{4 \cdot T_n}{f_N \cdot \varepsilon_n \cdot \pi \cdot \sqrt{(\Delta\nu \cdot \tau_n)}} \tag{4}$$

for the array of N antennas.

Equations (3)-(4) represent simplications of the the very complicated problem of surface brightness sensitivity for aperture synthesis arrays. For this reason let us discuss a simplified form of the comparision of surface brightness sensitivity for single antennas and packed arrays with filling factors (f_N) of the order of 0.3-0.5. The simplification that we will assume is that both antenna sensitivity patterns and u-v sampling for arrays can be approximated by simple gaussian functions.

The sensitivity pattern for a single antenna is then

$$P_{ant}(\theta, \theta_H) = \exp[-4 \cdot \ln(2) \cdot (\theta/\theta_H)^2] \tag{5}$$

where θ_H is the half-power beam width. The distribution of spatial frequencies sampled by an antenna with a gaussiam sensitivity pattern is given by the 2-D Fourier transform, which is a Hankel transform of P_{ant}. If we define $q = \sqrt{(u^2+v^2)}$ then the the (normalized) sensitivity in the spatial frequency domain is

$$n(q,\theta_H) = \exp\{-(\pi\cdot q\cdot\theta_H)^2/[4\cdot\ln(2)]\} \quad . \qquad (6)$$

In Figure 1 we plot $n_1(q) = n(q,\theta_1)$ and $n_n(q) = n(q,\theta_n)$, for a single large antenna with $D_1 = 128$ meters and the n-th array antenna with a diameter $D_n = 32$ meters, as a function of $q\cdot\lambda$. If we assume that the array consists of N = 16 antennas with a design giving a gaussian synthesized beam such that $\theta_B = \lambda/B_N$, where $B_N = 256$ meters,
then the spatial frequency sensitivity of the array is given by

$$n_B(q) = f_N\cdot n(q,\theta_B) = f_N\cdot\exp\{-(\pi\cdot q\cdot\theta_B)^2/[4\cdot\ln(2)]\} \quad \text{if} \quad q \geq 2/\theta_B$$
$$= 0 \quad \text{if} \quad q < 2/\theta_B. \qquad (7)$$

where f_N is the filling factor. This function is plotted (as filled triangles) in Figure 1 for a series of equally spaced sampling intervals. However, this distribution of sampling in spatial frequencies does not include the effects of the antenna beam distribution for each the antenna in the array. A correct simulation of the real situation would involve taking the discrete sampling of a source in the sky using $n_B(q)$, transforming the visibilities obtained back to the image domain, multiplying by the antenna beam for each array antenna, then transforming back to the spatial frequency domain. This is equivalent to convolving the discrete sampling distribution of the array (n_B) by the sampling distribution for each array antenna (n_n), as plotted in Figure 1 in the form of a one-dimensional approximation that we have labeled n_{Bconv}.

The point of the above discussion is that true surface brightness sensitivity is described by the n(q) distribution functions. Equations (3) and (4) are applicable to ONE of the the spatial frequencies being sampled - that which correspond to the n(q) = 1/2 point in a gaussian distribution. For the single antenna case this is well defined, but for the array case the sensitivity pattern $n_{Bconv}(q)$ ranges from 0 at q = 0 (the infamous "missing zero spacing"), to values increasing up to roughly the filling factor at $q \approx 2/\theta_n$, to values close to the idealized gaussian distribution for larger q. The differences between the curves in Figure 1 are important because they define the sense in which an array can or cannot "do the same" spatial frequency sampling as does a large single antenna. The complete comparison of a packed array with a large single dish involves comparing the $n_1(q)$ curve with BOTH the $n_{Bconv}(q)$ for visibility data obtained with the array and the $n_n(q)$ for "single dish data" obtained from the array antennas. The latter is obtained by either making use of auto-correlation data or using the antennas in "single dish" observing modes. The near-equivalence to the large single antenna case is obtained ONLY when the array is used in both aperture synthesis mode and one of the two modes of sampling the "zero" spacing.

Having discussed the special problem of comparing surface brightness sensitivity for single antennas and arrays, let us return to the simple analysis based up Equation (1)-(4) where, for surface brightness, only one "main" spatial frequency of sampling is considered.

2.3 Single Field Problems

When the scientific problem requires a certain minimum noise level, one simply solves Equations (1)-(4) for the required observing time for that field, i.e.

$$\tau_1 = \frac{64\cdot(k_B\cdot T_1)^2}{(\pi\cdot D_1^2\cdot\varepsilon_1\cdot\Delta S_1)^2\cdot\Delta\nu} = \frac{16\cdot T_1^2}{(\pi\cdot\varepsilon_1\cdot\Delta T_1)^2\cdot\Delta\nu} \tag{8}$$

for the case of a large single dish, and

$$\tau_N = \frac{64\cdot(k_B\cdot T_N)^2}{(\pi\cdot N\cdot D_N^2\cdot\varepsilon_N\cdot\Delta S_N)^2\cdot\Delta\nu} = \frac{16\cdot T_N^2}{(\pi\cdot f_N\varepsilon_N\cdot\Delta T_N)^2\cdot\Delta\nu} \tag{9}$$

for the case of an array of N antennas. The relative speed of a single dish compared to an array, for point source detection problems in a single field, is then

$$\tau_1/\tau_N = (\varepsilon_n/\varepsilon_1)^2\cdot(T_1/T_N)^2\cdot N^2\cdot(D_N/D_1)^4 \tag{10}$$

and for surface brightness measurements of a single field

$$\tau_1/\tau_N = (\varepsilon_n/\varepsilon_1)^2\cdot(T_1/T_N)^2\cdot f_N^{\,2} \quad . \tag{11}$$

If one poses the problem in terms of a single dish vs an array with the same collecting area, then $N\cdot D_n^2 = D_1^2$, and Equations (10)-(11) become

$$\tau_1/\tau_N = (\varepsilon_n/\varepsilon_1)^2\cdot(T_1/T_N)^2 \tag{12}$$

and

$$\tau_1/\tau_N = (\varepsilon_n/\varepsilon_1)^2 \cdot (T_1/T_N)^2 \cdot f_N^{\,2} \quad . \tag{13}$$

Based upon experience with packed arrays studied as part of the Millimeter Array design, it should be possible to attain $f_N = 0.4$.

Equations (10) and (12) show that, for single field, point source problems the single dish and array are comparable if efficiencies, system temperatures, and collecting areas are comparable; however, if, as is current practice, single dish receivers have lower system temperatures, then the advantage is with the single dish. Equations (11) and (13) further illustrate the well-known fact that unfilled aperture instruments are poorer at surface brightness measurements, making this one of the strongest advantages of the single dish approach.

2.4 Large Field Problems

When the scientific problem requires that one image a source with a solid angle Ω_s, which is much larger than the field of a single antenna beam, the number of required antenna beam pointings affects the total integration time needed. The source integration time, for sampling two points per beam, then becomes

$$\tau_1^s = \frac{4 \cdot \Omega_s \cdot D_1^2 \cdot \tau_1}{\lambda^2 \cdot N_{beams}} \tag{14}$$

for a single dish multi-beamed with N_{beams}, and

$$\tau_N^s = \frac{4 \cdot \Omega_s \cdot D_N^2 \cdot \tau_N}{\lambda^2} \tag{15}$$

for an array.

The relative speed at which instruments achieve a specified noise level in Ω_s is then

$$\tau_1^s/\tau_N^s = (\varepsilon_n/\varepsilon_1)^2 \cdot (T_1/T_N)^2 \cdot N^2 \cdot (D_N/D_1)^2/N_{beams} \tag{16}$$

and for surface brightness measurements

$$\tau_1^s/\tau_N^s = (\varepsilon_n/\varepsilon_1)^2 \cdot (T_1/T_N)^2 \cdot N^2 \cdot (D_1/D_n)^2 \cdot f_N^{\,2}/N_{beams} \quad . \tag{17}$$

If one poses the problem in terms of a single dish vs an array with the same collecting area, then $N \cdot D_n^2 = D_1^2$, so Equations (16)–(17) become, for the large source and point source cases,

$$\tau_1^s/\tau_N^s = (\varepsilon_n/\varepsilon_1)^2 \cdot (T_1/T_N)^2 \cdot N/N_{beams} \tag{18}$$

and for large source and surface brightness measurements

$$\tau_1^s/\tau_N^s = (\varepsilon_n/\varepsilon_1)^2 \cdot (T_1/T_N)^2 \cdot f_N{}^2 \cdot N/N_{beams} \quad . \tag{19}$$

Equations (16)–(19) show that unless one has N_{beams} of the order of N, the advantages for large field problems shifts to the packed arrays because of the larger antenna beams.

2.5 Source Confusion

There is another factor that limits the effectiveness of the single dish approach – the well known source confusion problem. There is some point when increased integration time brings the rms noise level down to a level where there is a high probability of multiple sources in the antenna beam. For the single dish approach that strongly limits the flux level to which observations can be made for many scientific problems. However, for arrays used for aperture synthesis imaging, the confusion limit is set by the synthesized beam size which is much smaller. This means that much work at the lower flux levels is severely limited with single dishes and done more advantageously with arrays.

2.6 Other Limitations on Integration Time

It is well know that many single dishes are severely limited in useful integration time. For spectroscopic work this is not important, but it is a major limitation for continuum work. This limitation occurs whenever the antenna/feed "sees" other systematic sources of radio emission besides the radio sky and atmosphere. "Clean" designs for antenna and optics should be capable of reducing the effects of this problem. Packed arrays are not significantly affected by this problem when operating as correlation arrays.

3.0 Conclusions

In the final analysis the choice between a single dish and a packed array approach, where both have comparable collecting area, depends upon the emphasis upon which scientific problems are the primary goal. For point source, single field problems (where the confusion limit is not a factor) there is a slight edge for the single dish because system temperatures are likely to be lower. This factor also means less collecting area in a single dish may be chosen without giving the advantage back to the arrays. For large field, point source problems the packed arrays will have the speed advantage unless the single dish is multi-beamed to the point where $N/N_{beams} \approx 1$. Emphasis on problems measuring surface brightness introduces a factor of f_N^2 advantage for the single dish approach. On the other hand, weak source problems are fundamentally limited by source confusion in the single dish approach.

In this discussion we have not mentioned the operational aspects of the single dish vs the packed array approach. There is no doubt that arrays are more expensive to operate. There is also no doubt that the computing problems for arrays are more expensive to solve.

The objective of this memo is to quantitatively analyze the speed advantages for single dishes vs packed arrays. The answer is ambiguous in the sense that the advantage goes to one concept or the other depending upon the problem being posed. This, and the operational factors, make the final choice primarily dependent upon a combination scientific choices and technical/budgetary considerations.

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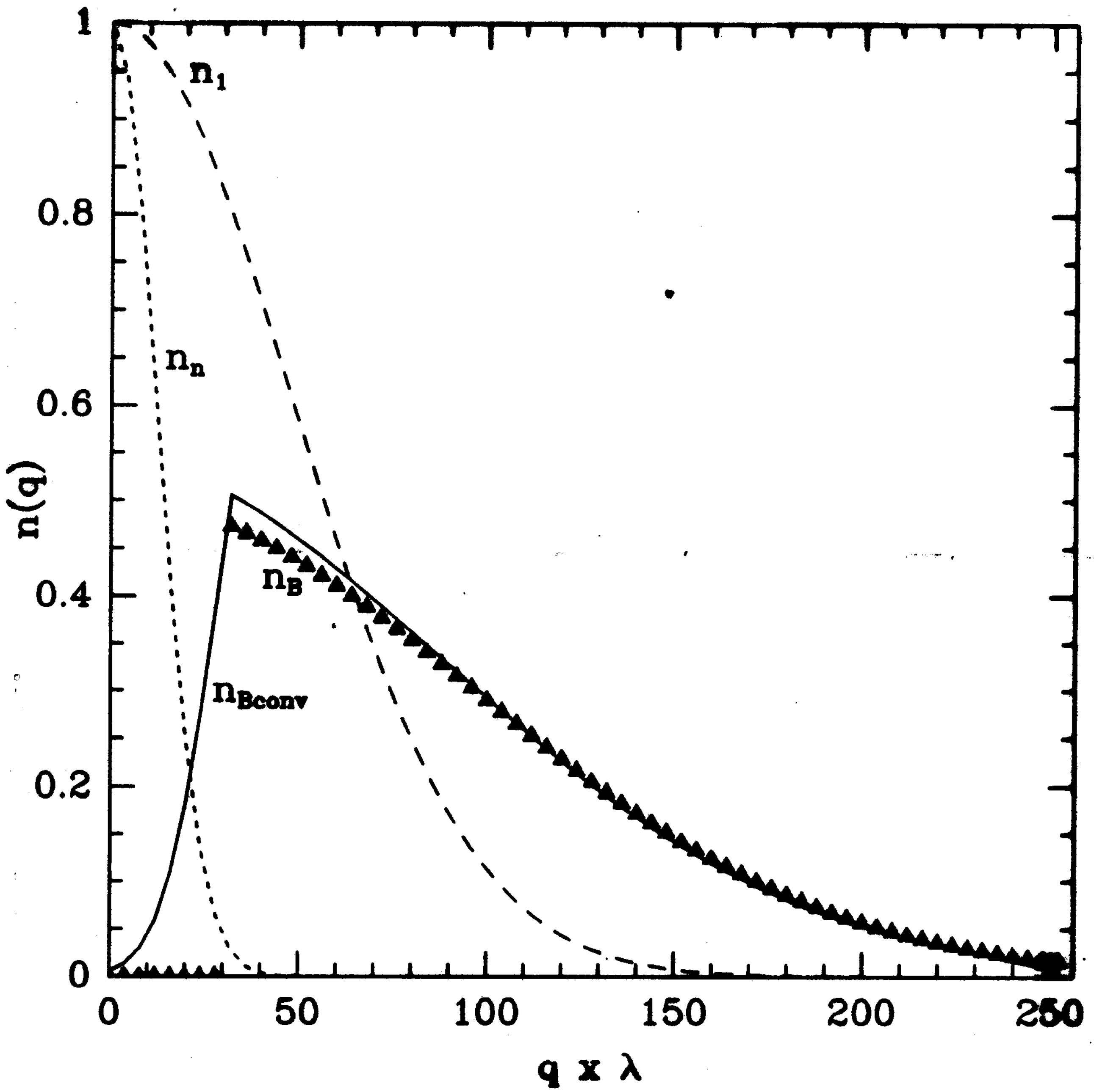

Figure 1 – The normalized spatial frequency sampling function, n(q), is plotted as a funtion of q•λ for: (n_1) a single dish with D_1 = 128 meters; (n_n) the nth single dish (of diameter D_n = 32 meters) in a array; (n_B) discrete sampling for an array; and (n_{Bconv}) the effective sampling for an array.