MILLIMETER ARRAY
MEMO NO. 35

Factors Affecting the Sensitivity of a Millimeter Array -- Further Discussion

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Millimeter Array Memo No. 33 (R. M. Hjellming) detailed the general effects of atmospheric absorption and emission on the sensitivity of a millimeter wave aperture synthesis array. Telescope losses were accounted for by a separate aperture efficiency factor appearing in the sensitivity equation. Over the past several years, the treatment of sensitivity, calibration, and losses for single dish millimeter wave astronomy has evolved away from the traditional approaches at longer wavelengths. Many of the single dish millimeter wave concepts are pertinent to the aperture synthesis case as well. This memo outlines these concepts, which are based largely upon the work of Ulich and Haas (1976), Kutner and Ulich (1981), and references therein.

In general, antenna losses fall into one of the following categories: ohmic losses in reflective surfaces, aperture blockage, illumination spillover past reflectors, and surface imperfections which reduce the power in the main diffraction beam. Traditionally, these losses have been lumped into simple aperture or beam efficiency factors. In the millimeter wave regime, a distinction should be made between antenna power that is terminated at ambient temperatures on the ground or on antenna structures and antenna power that is terminated on the sky. As will be shown below, power terminated on ambient temperature blackbodies affects noise and sensitivity differently than that terminated on the sky.

Antenna losses that appear as ambient temperature terminations arise from ohmic heating, described by the efficiency η_r, and rear spillover and scattering, described by the efficiency η_{rss}. Formally, η_r is defined as

$$\eta_r \equiv (G/4\pi)\iint_{4\pi} P_n(\Omega)\,d\Omega, \tag{1}$$

where G is the maximum antenna gain,

P_n is the antenna power pattern, normalized so that

$P_n(\Omega)=1$, and

Ω is solid angle.

The formal definition of η_{rss} is

$$\eta_{rss} \equiv \iint_{2\pi} P_n(\Omega)\,d\Omega \Big/ \iint_{4\pi} P_n(\Omega)\,d\Omega, \tag{2}$$

where the integration in the numerator is over the forward hemisphere. The efficiency factor η_ℓ, given by

$$\eta_\ell \equiv \eta_r \eta_{rss} \tag{3}$$

accounts for all losses appearing as ambient temperature terminations, and is sometimes called the "warm spillover efficiency." η_ℓ thus represents the fraction of total antenna power response that falls on the forward hemisphere.

Most telescopes are also affected by forward spillover and scattering losses in which power is received from the sky but not in the region of the main diffraction beam. This loss is described by the efficiency factor η_{fss}, which is sometimes called the "cold spillover efficiency." Figure 1 illustrates how rear and forward spillover arise in an antenna with Cassegrain optics. The magnitudes of η_ℓ and η_{fss} depend upon the type of optics employed on the antenna, the illumination pattern of the feeds, the amount of aperture blockage, and other factors. Typically, each efficiency factor will be within the range 0.8 - 0.9.

As part of the definition of η_{fss}, the forward hemisphere must be divided into a diffraction zone Ω_d centered on the main beam and a spillover zone which is the remainder of the forward

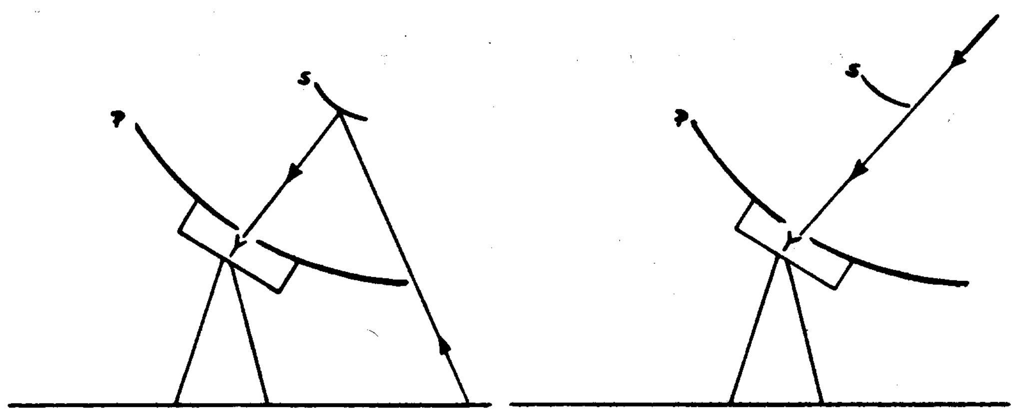

Figure 1 -- Rear and forward spillover for an antenna with Cassegrain optics. The primary and secondary reflectors are denoted by "P" and "S," respectively.

hemisphere. Formally,

$$\eta_{fss} \equiv \iint_{\Omega_d} P_n(\Omega)\,d\Omega \Big/ \iint_{2\pi} P_n(\Omega)\,d\Omega, \tag{4}$$

where the integral over 2π again refers to the forward hemisphere. The size of Ω_d can be defined arbitrarily. In the single dish case, it is usually defined to be sufficiently large to contain the majority of the power in the error pattern. If the error pattern is unimportant, Ω_d might be defined to include only the main diffraction beam. Hence, η_{fss} is the fraction of the antenna power on the forward hemisphere that falls within Ω_d. The fraction of power within Ω_d that is concentrated in the main beam is denoted by η_m^*. This efficiency factor accounts for error pattern and sidelobe losses; if the error pattern or sidelobes couple with the source, η_m^* is of limited use as a calibration factor. Figure 2 illustrates the relation between η_ℓ, η_{fss}, and η_m^*. The conventional main beam efficiency η_m is related to the efficiencies defined above by

$$\eta_m = \eta_\ell \eta_{fss} \eta_m^*. \tag{5}$$

The efficiency with which a single beam couples with the source is given by

$$\eta_c = \iint_{\Omega_s} P_n(\psi-\Omega) B_n(\psi)\,d\psi \Big/ \iint_{\Omega_d} P_n(\Omega)\,d\Omega, \tag{6}$$

where Ω_s is the solid angle of the source,

B_n is the normalized brightness temperature of the source,

and

ψ is the direction angle on the sky.

If T_R is the radiation temperature of the source, then

$$T_R^* = \eta_c T_R. \tag{7}$$

T_R^* is, thus, the source antenna temperature corrected for all telescope and atmospheric losses except for source-beam coupling.

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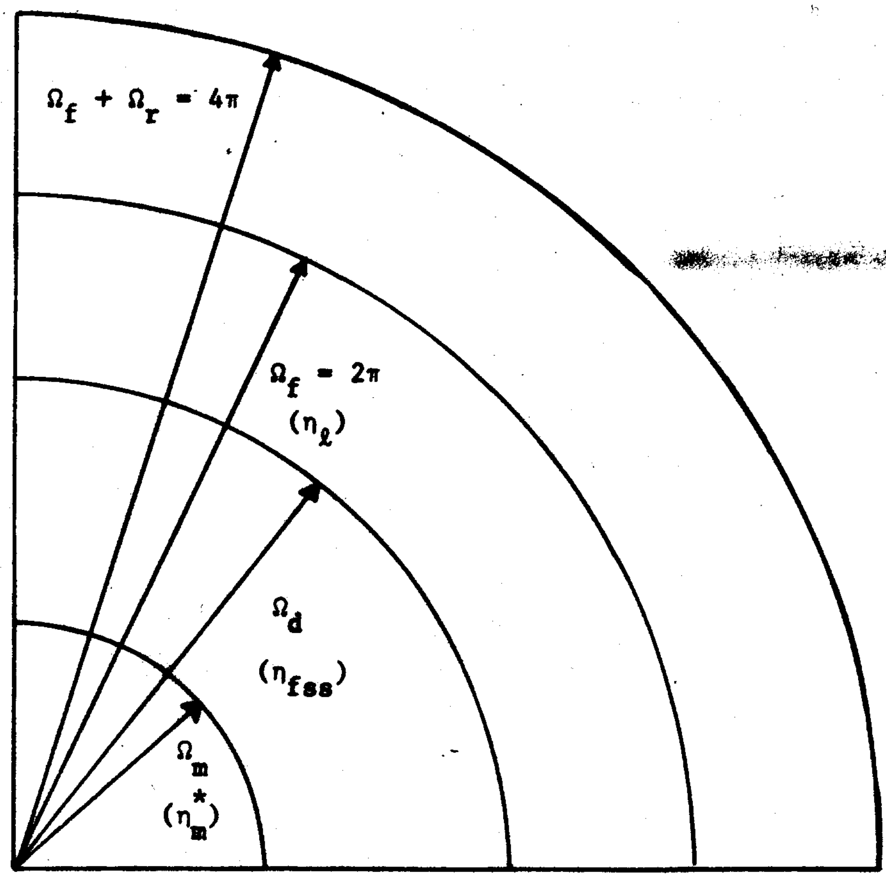

Figure 2 -- The relationship between the solid angle zones discussed in this memo (Ω_f is the foward hemisphere, Ω_r is the rear hemisphere; all other symbols are defined in the text). The efficiency factors in parentheses represent the fraction of antenna response in the associated solid angle relative to the solid angle that encompasses it.

Given the efficiency factors defined above, an effective system temperature T^*_{sys} can be derived that includes the effects of atmospheric absorption and emission and terms for all telescope losses currently recognized. The most straightforward definition of effective system temperature comes from the single dish relation

$$T^*_R = \frac{T_A^{SOURCE} - T_A^{REF}}{T_A^{REF}} T^*_{sys}, \tag{8}$$

where T_A^{SOURCE} is the total power antenna temperature with the telescope on the source position, and

T_A^{REF} is the total power antenna temperature with the telescope on a blank sky reference position.

Both internal receiver noise and external emission from the sky contribute to the total power antenna temperatures. Thus,

$$T_A^{REF} = T_{RX} + T_{sky}^{REF}, \tag{9}$$

where T_{RX} is the receiver noise temperature (as might be measured by a hot/cold load calibration), and

T_{sky}^{REF} is the emission from blank sky (including rear spillover, blockage, and ohmic loss contributions).

Whether the Millimeter Array will always operate in a pure, single sideband mode, in a double sideband mode, or with optional image rejection is as yet undetermined. Formalism for the general case of double sideband observations with unequal gains in the two sidebands is used here. Let G_s and G_i be the gains of the signal and image sidebands, respectively, normalized so that $G_s + G_i = 1$. The contribution of the two sidebands to the total receiver noise temperature can be explicitly denoted by rewriting T_{RX} as

$$T_{RX} = (G_s + G_i)T_{RX}. \qquad (10)$$

The sky temperature has several components, each of which is characterized by a blackbody temperature. In the millimeter regime, the Rayleigh-Jeans approximation to the blackbody spectrum breaks down. We define a Rayleigh-Jeans equivalent temperature that characterizes the emission of a blackbody of temperature T and frequency ν as

$$R(\nu,T) = \frac{h\nu/k}{\exp(h\nu/kT) - 1}. \qquad (11)$$

The sky temperature in the reference position is given by

$$T_{sky}^{REF} = G_s[\eta_\ell R(\nu_s,T_s) + (1 - \eta_\ell)R(\nu_s,T_{sbr})]$$
$$+ G_i[\eta_\ell R(\nu_i,T_i) + (1 - \eta_\ell)R(\nu_i,T_{sbr})], \qquad (12)$$

where ν_s is the signal frequency,

ν_i is the corresponding image frequency, and

T_{sbr} is the received temperature resulting from rear spillover, blockage, and ohmic losses.

$R(\nu,T_s)$ is the effective temperature of the sky in the signal sideband and is given by

$$R(\nu_s,T_s) = R(\nu_s,T_{atm})[1 - \exp(-\tau_s A)] + R(\nu_s,T_{bg})\exp(-\tau_s A), \qquad (13)$$

where τ_s is the zenith optical depth of the atmosphere at the signal frequency,

T_{atm} is the mean temperature of the atmosphere,

A is the number of airmasses at the observing position, and

T_{bg} is the temperature of the cosmic background radiation.

Similarly, the effective temperature of the sky in the image sideband is

$$R(\nu_i,T_i) = R(\nu_i,T_{atm})[1 - \exp(-\tau_i A)] + R(\nu_i,T_{bg})\exp(-\tau_i A) \qquad (14)$$

where τ_i is the atmospheric zenith optical depth at the image frequency. For first I.F. frequencies of a few GHz, $R(\nu_s,T_{bg}) = R(\nu_i,T_{bg})$ and $R(\nu_s,T_{sbr}) = R(\nu_i,T_{sbr})$. At most frequencies, $\tau_s = \tau_i$ and $R(\nu_s,T_{atm}) = R(\nu_i,T_{atm})$. These last approximations are not true for some frequencies, such as 115 GHz (the CO J = 1 → 0 line) that are in the wings of atmospheric O_2 and H_2O lines.

The effective system temperature T^*_{sys} differs between the spectral line case, in which the signal originates in only one sideband, and the continuum case, in which signal is received in both sidebands. Only the spectral line case is considered here.

The expression for T_A^{SOURCE} can be divided between the beam components that are coupled with the source and those that are coupled with cold sky:

$$T_A^{SOURCE} = (G_s + G_i)T_{RX} + G_s[\eta_\ell(1 - \eta_{fss}\eta_c)R(\nu_s,T_s)$$
$$+ \eta_\ell\eta_{fss}\eta_c\{R(\nu_s,T_E)[1 - \exp(-\tau)]\exp(-\tau_s A)$$
$$+ R(\nu_s,T_{bg})\exp(-\tau)\exp(-\tau_s A) + R(\nu_s,T_{atm})[1 - \exp(-\tau_s A)]\}$$
$$+ (1 - \eta_\ell)R(\nu_s,T_{sbr})] + G_i[\eta_\ell R(\nu_i,T_i) + (1 - \eta_\ell)R(\nu_i,T_{sbr})], \tag{15}$$

where τ is the optical depth of the source, and

T_E is the excitation temperature of the source.

The effect of forward spillover is handled only approximately by the expression above, since forward spillover may fall diffusely on the sky and is not associated with a specific airmass.

The source radiation temperature (excess line brightness temperature) is defined as

$$T_R = [R(\nu_s,T_E) - R(\nu_s,T_{bg})](1 - e^{-\tau}). \tag{16}$$

This relation may be used to write the term in {} in Equation (15) as

$$T_R \exp(-\tau_s A) + R(\nu_s,T_{bg})\exp(-\tau_s A) + R(\nu_s,T_{atm})[1 - \exp(-\tau_s A)].$$

Hence,

$$T_A^{SOURCE} - T_A^{REF} = G_s\eta_\ell\eta_{fss}T_R^* \exp(-\tau_s A), \tag{17}$$

where T_R^* is given in Equation (7). Finally, from the defining relation of Equation (8),

$$T_{sys}^* = \frac{[1 + (G_i/G_s)]T_{RX} + (1/G_s)T_{sky}^{REF}}{\eta_\ell \eta_{fss} \exp(-\tau_s A)}, \quad (18)$$

where T_{sky}^{REF} is defined in Equation (12). The numerator of Equation (18) shows how T_{sys}^* is affected by receiver noise and atmospheric emission; the denominator shows how T_{sys}^* is affected by antenna losses and atmospheric absorption.

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Consider the following case as an example:

$G_s = 1$, $G_i = 0$ (pure single sideband)
$T_{RX}(SSB) = 100$ K
$\nu_s = 230.5$ GHz
$\tau_s = 0.2$
$A = 2$ ($A = \sec(z)$, 30° elevation)
$\eta_\ell = 0.85$
$\eta_{fss} = 0.85$
$T_{atm} = 280$ K
$T_{sbr} = 280$ K
$T_{bg} = 2.7$ K

From Equation (12),

$$T_{sky}^{REF}(SSB) = \eta_\ell R(\nu_s, T_{atm})[1 - \exp(-\tau_s A)] \qquad (= 76.9)$$

$$+ R(\nu_s, T_{bg})\exp(-\tau_s A) \qquad (= 0.1)$$

$$+ (1 - \eta_\ell)R(\nu_s, T_{sbr}) \qquad (= 41.2)$$

$$= 118.2$$

From Equation (18),

$$T_{sys}^{*} = \frac{100 + 118.2}{(0.85)(0.85)(0.67)} = 451 \text{ K}$$

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REFERENCES

Kutner, M. L., and Ulich, B. L., 1981, Ap. J., 250, 341.

Ulich, B. L., and Haas, R. W., 1976, Ap. J. Suppl., 30, 247.